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Determination of the optimal allocation of parameters for gauge repeatability and reproducibility study

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Abstract *Recently, gauge repeatability and reproducibility (GR&R) study has been highly regarded by the quality practitioners when QS9000 and D19000 become fashionable requirements for manufacturing industries. Measurement plays a significant role in helping organizations improve their product quality. Good quality of products is the key factor towards business success. Therefore, how to ensure the quality of measurement becomes an important task for quality practitioners. In performing the GR&R study, several parameters, such as the appropriate sample size of parts* (n)*, number of inspectors* (p) *and replicate measurements* (k) *are frequently asked by quality personnel in industries. The adequacy of current way of* (n, p, k) *selection is very questionable. A statistical method using the shortest confidence interval and its associated computer programming algorithm are presented in this paper for evaluating the optimal allocation among sample size of parts* (n)*, number of inspectors* (p) *and replicate measurements* (k)*. Hopefully, it can provide a useful reference for quality practitioners in industries.*

1. Introduction

Gauge variability plays a key role on quality improvement for the industry. Only a gauge with acceptable repeatability and reproducibility, the adequacy of a product's measurement process can be determined. In many companies, the requirement of having a sound measurement system is part of their total quality assurance programs. Good quality of product can only be achieved through an adequate measurement system. Therefore, making sure the performance of a measurement system is adequate becomes an urgent task for practitioners. Recently, quality practitioners have paid more attention to the gauge repeatability and reproducibility (GR&R) analysis. GR&R is part of the requirements for QS9000 initiated by the Ford Company and widely accepted by auto manufacturers afterwards. Prior to the development of QS9000, the three major auto manufacturers in USA had their own quality systems. In order to adapt for the trend of ISO and become an international quality standard for the auto suppliers, QS9000 and one of the six handbooks of QS9000, Measurement System Analysis (MSA), have been developed accordingly.

Generally speaking, the GR&R study is performed according to the QS9000 standards stated in MSA.1. to decide the suitability of a gauge. Prior to performing the GR&R study, one should determine sample size of parts (n), number of inspectors (p), and replicate measurements (k). As the total number of measurements increase, the

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estimated total variation becomes more precise, but the related cost and time will be increased too. Although $n = 10, 15, 20$; $p = 2, 3$ and $k = 2, 3$ are commonly used (n, p, k) combinations in most industries, the adequacy/accuracy of any of these combinations of GR&R parameters (i.e. sample size of parts n, number of inspectors p, number of measurements k) is very questionable. Therefore, the objectives of this research is to determine an optimal allocation among sample size of parts n, number of inspectors p, and repeated measurements k with a given fixed cost of total measurement number or (n, p, k) combination.

2. Literature review

Generally speaking, there are two sources influencing gauge precision and accuracy (Burdick and Larsen, 1997; Montgomery and Runger, 1993a, b; Pan and Lee, 2003; Tsai, 1989):

(1) *Gauge error*. When an inspector uses the same gauge to measure a product several times under the same conditions, then several different values of measurement may occur. This error, called repeatability, comes from the gauge itself.

(2) *Inspector error*. This error occurs when different inspectors measure a product under the same condition and is called reproducibility. This error occurs when inspectors do not get sufficient training or inspectors do not measure a product according to standard procedure. The variability comes from the inspectors.

Therefore, the variability of measurement process can be defined as:

$$\sigma^2_{\text{gauge}} = \sigma^2_{\text{repeatability}} + \sigma^2_{\text{reproducibility}}, \tag{2-1}$$

where $\sigma^2_{\text{gauge}}$is the variability of measurement process, $\sigma^2_{\text{repeatability}}$ the repeatability, $\sigma^2_{\text{reproducibility}}$ and the reproducibility. Total variation is the sum of product variation and the variability of measurement process:

$$\sigma^2_{\text{Total}} = \sigma^2_{\text{part}} + \sigma^2_{\text{gauge}}, \tag{2-2}$$

where σ^2_{Total} is total variation, σ^2_{part} the product variation, σ^2_{gauge} the variability of measurement process or gauge.

According to Tsai's (1989) ANOVA model, this research is a two-factor design of experiment under the same condition of measurement, where one factor is the inspector, the other factor is the product, and both are the interaction and random effect. The model is listed as follows:

$$y_{ijl} = \mu + P_i + O_j + (PO)_{ij} + R_{ijl} \begin{Bmatrix} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, p \\ l = 1, 2, \ldots, k \end{Bmatrix} \tag{2-3}$$

where:

μ = measurement mean (total mean).

P_i = effect of product (random effect).

O_j = effect of inspector (random effect).

$(PO)_{ij}$ = effect of interaction between product and inspector (random effect).

R_{ijl} = effect of replicate measurements (error term).

By the result of measurements and ANOVA method, one can obtain an ANOVA table as shown in Table I.

From Table I, the sum of square, degree of freedom, mean square, and expected mean square of sources of variability, where expected mean square includes variation between product and inspector, and error term. By using the four expected mean squares in Table I, one can get the estimated values of these sources of variation, which are shown below:

$$
\begin{aligned}
\hat{\sigma}_R^2 &= \mathrm{MS}_R \\
\hat{\sigma}_{PO}^2 &= (\mathrm{MS}_{PO} - \mathrm{MS}_R)/k \\
\hat{\sigma}_P^2 &= (\mathrm{MS}_P - \mathrm{MS}_{PO})/pk \\
\hat{\sigma}_O^2 &= (\mathrm{MS}_O - \mathrm{MS}_{PO})/nk.
\end{aligned}
\tag{2-4}
$$

Then the repeatability, reproducibility, and the variability of gauge can be calculated through the following:

$$\hat{\sigma}^2_{\text{repeatability}} = \hat{\sigma}_R^2 = \mathrm{MS}_R$$

$$\hat{\sigma}^2_{\text{reproducibility}} = \hat{\sigma}_O^2 + \hat{\sigma}_{PO}^2 = (\mathrm{MS}_O + (n-1)\mathrm{MS}_{PO} - nMS_R)/nk$$

$$\hat{\sigma}^2_{\text{gauge}} = \hat{\sigma}^2_{\text{repeatability}} + \hat{\sigma}^2_{\text{reproducibility}} = (\mathrm{MS}_O + (n-1)\mathrm{MS}_{PO} + n(k-1)\mathrm{MS}_R)/nk.$$

If interaction between product and inspector does not exist, then sum of square, degree of freedom, and mean square should be added to error term.

Montgomery and Runger (1993a, b) proposed another method: "Classical Gauge Repeatability and Reproducibility Study," which used the idea of mean and range, i.e. $\bar{\bar{R}}/d_2$ and $R_{\bar{\bar{x}}}/d_2$(d_2 is the adjustment factor shown in Table II) to find the variability of measurement process. This method first estimates the repeatability by $\bar{\bar{R}}/d_2$ as shown in equations (2-5) and (2-6):

Table I.
ANOVA table of two-factor design of experiment

Source of variability	Sum of square	Degrees of freedom	Mean of square	Expected mean square
Parts	SS_P	$n_p = n - 1$	MS_P	$E(\mathrm{MS}_P) = \sigma_R^2 + k\sigma_{PO}^2 + pk\sigma_P^2$
Inspector	SS_O	$n_o = p - 1$	MS_O	$E(\mathrm{MS}_O) = \sigma_R^2 + k\sigma_{PO}^2 + nk\sigma_O^2$
Parts*inspector	SS_{PO}	$n_{po} = (n-1)(p-1)$	MS_{PO}	$E(\mathrm{MS}_{PO}) = \sigma_R^2 + k\sigma_{PO}^2$
Error	SS_R	$n_R = np(k-1)$	MS_R	$E(\mathrm{MS}_R) = \sigma_R^2$
Total	SS_T	$npk - 1$		

k	m 1	2	3	4	5	8	10	∞
2	0.709	0.781	0.813	0.826	0.840	0.855	0.862	0.885
3	0.524	0.552	0.565	0.571	0.575	0.581	0.581	0.592
4	0.446	0.465	0.472	0.474	0.476	0.481	0.481	0.485
5	0.403	0.417	0.420	0.422	0.424	0.426	0.427	0.429
6	0.375	0.385	0.388	0.389	0.391	0.392	0.392	0.395
7	0.353	0.361	0.364	0.365	0.366	0.368	0.368	0.370
8	0.338	0.344	0.346	0.347	0.348	0.348	0.350	0.351
9	0.325	0.331	0.332	0.333	0.334	0.334	0.336	0.337
10	0.314	0.319	0.322	0.323	0.323	0.324	0.324	0.325

Notes: m = sample size $\times$ number of inspectors; k = replicate measurements

Table II. The coefficient of adjustment $\frac{1}{d_2}$ under different combination of m and k6

$$\bar{\bar{R}} = \frac{\sum_{j=1}^{p} \bar{R}_j}{p} \tag{2-5}$$

$$\hat{\sigma}_{\text{repeatability}} = \frac{\bar{\bar{R}}}{d_2} \tag{2-6}$$

where $\bar{R}_j$ is the range of repeated measurements averaged across parts within by the jth inspector, and d_2 can be found in Table II. Then the estimate of repeatability $R_{\bar{\bar{x}}}/d_2$ can be obtained by equations (2-7) and (2-8).

$$R_{\bar{\bar{x}}} = \bar{\bar{x}}_{\max} - \bar{\bar{x}}_{\min}, \tag{2-7}$$

$$\hat{\sigma}_{\text{reproducibility}} = \frac{R_{\bar{\bar{x}}}}{d_2}, \tag{2-8}$$

where $\bar{\bar{x}}$ is the mean of sample average consisting of replicate data obtained by one inspector/operator. $\bar{\bar{x}}_{\max}$ is the maximum of $\bar{\bar{x}}$ between inspectors. Similarly, $\bar{\bar{x}}_{\min}$ is the minimum of $\bar{\bar{x}}$ between inspectors, d_2 could be found in Table II. By equations (2-6), (2-8), the variability of measurement process can be defined as:

$$\hat{\sigma}^2_{\text{gauge}} = \hat{\sigma}^2_{\text{repeatability}} + \hat{\sigma}^2_{\text{reproducibility}}. \tag{2-9}$$

The condition to use Classical GR&R to estimate repeatability and reproducibility is that all $\bar{R}_j$ fall within the control limits of R chart for ensuring the stability to assess the measurement system.

AIAG (1997, 1995) and DataMyte Editing Group (1989) state a method called Long Form, which is a standard form designed by three major automobile manufacturers in the USA. The form uses sample range method to estimate repeatability and reproducibility, is primarily designed for quality practitioners without statistical background. The GR&R and whether the measuring system is suitable can be determined through step-by-step procedures. The data can be gathered by inspectors, and then put into a standard format, thus the repeatability, reproducibility, and product variation can be easily estimated.

3. Design and selection of optimal allocation of GR&R parameters

In order to determine the optimal allocation among sample size of parts (n), number of inspectors (p) and replications (k), we use the ANOVA random effect model of two-factor design of experiment as shown in equation (2-3) to minimize the widths of confidence interval under various combinations of n, p and k. The combination with shortest width of confidence interval is then considered to be optimal (Montgomeryand Runger, 1993a, b).

A computer programming algorithm is also developed to search for the optimal combinations of n, p and k. The following are the procedures for searching the optimal (n, p, k) combination.

Step 1

Consider various combinations of n, p and k. Let sample size of parts (n) be 5, 10, 15, and 20, number of inspectors (p) be 2, 3, 4, repeated measurements (k) be 2, 3, 4. Thus, there are 36 (n, p, k) combinations as shown in Table III.

Table III implies that among those 36 (n, p, k) combinations, some of them have the same total measurement number. For example, with a total measurement number 80, there are four (n, p, k) combinations, i.e. (5, 4, 4), (10, 2, 4), (10, 4, 2), and (20, 2, 2). Thus, the widths of confidence interval of measurement variability for different combinations can be compared under the same total measurement number.

Step 2

Choose various values of sources of variations σ_O^2, σ_{PO}^2, and σ_R^2 to start computer simulation:

- without loss of generality, we set variation due to inspector, $\sigma_O^2 = 0.5$, 1.0, or 1.5, i.e. $\sigma_O^2 \in \{0.5, 1.0, 1.5\}$;
- similarly, we set variation due to interaction between inspector and product, $\sigma_{PO}^2 = 0.5$, 1.0, or 1.5, i.e. $\sigma_{PO}^2 \in \{0.5, 1.0, 1.5\}$; and
- set variation due to error term, $\sigma_R^2 = 0.5$, 1.0, or 1.5, i.e. $\sigma_R^2 \in \{0.5, 1.0, 1.5\}$.

Step 3

Define the expected mean square as shown in equation (3-1). Substitute the (n, p, k) combination of Step 1 and values of σ_O^2, σ_{PO}^2, σ_R^2 of Step 2 to the expected mean square in equation (3-1), then the estimated mean square θ_O, θ_{PO}, and θ_R can be obtained:

Table III. Various combinations of sample size of parts, number of inspectors and repeated measurements

Number of inspectors p	Replicate measurements k	Sample size of parts n: 5	10	15	20
2	2	20	40	60	80
2	3	30	60	90	120
2	4	40	80	120	160
3	2	30	60	90	120
3	3	45	90	135	180
3	4	60	120	180	240
4	2	40	80	120	160
4	3	60	120	180	240
4	4	80	160	240	320

$$\begin{aligned}
\theta_O &= E(\mathrm{MS}_O) = \sigma_R^2 + k\sigma_{PO}^2 + nk\sigma_O^2 \\
\theta_{PO} &= E(\mathrm{MS}_{PO}) = \sigma_R^2 + k\sigma_{PO}^2 \\
\theta_R &= E(\mathrm{MS}_R) = \sigma_R^2.
\end{aligned} \tag{3-1}$$

Step 4

Use the (n, p, k) combination obtained in Step 1 and the expected mean square θ_O, θ_{PO}, and θ_R obtained in Step 3 to simulate chi-square random variables including $n_O\mathrm{MS}_O/\theta_O$, $n_{PO}\mathrm{MS}_{PO}/\theta_{PO}$ and $n_R\mathrm{MS}_R/\theta_R$, where n_O is $(n-1)$,n_{PO} is $(n-1)(p-1)$, n_R is $np(k-1)$. Thus, the estimated mean square MS_O, MS_{PO}, and MS_R can be obtained.

Step 5

Substitute the (n, p, k) combination from Step 1 and substitute the expected mean square MS_O, MS_{PO}, MS_R from Step 4 to the confidence interval of measurement variability. Then one can obtain the width of confidence interval of measurement variability, where confidence interval of measurement variability is obtained by using the approximated $100(1-\alpha)\%$ confidence interval of σ^2_{gauge} shown as below. This is derived from Montgomery and Runger (1993a, b):

$$\frac{u\hat{\sigma}^2_{\text{gauge}}}{\chi^2_{\alpha/2,u}} \leq \sigma^2_{\text{gauge}} \leq \frac{u\hat{\sigma}^2_{\text{gauge}}}{\chi^2_{1-\alpha/2,u}}, \tag{3-2}$$

where:

$$u = \left(\hat{\sigma}^2_{\text{gauge}}\right)^2 \left[\frac{(1/np)^2 MS_O^2}{o-1} + \frac{[(p-1)/np]^2 MS_{OP}^2}{(o-1)(p-1)} + \frac{[(n-1)/n]^2 MS_R^2}{op(n-1)}\right]^{-1}.$$

If $\hat{\sigma}^2_{po}$, variation of interaction between product and inspector, is not significant, then the variation of interaction should be incorporated into the error term, therefore the approximated $100(1-\alpha)\%$ confidence interval of measurement variability can be expressed as below:

$$\frac{w\hat{\sigma}^2_{\text{gauge}}}{\chi^2_{\alpha/2,u}} \leq \sigma^2_{\text{gauge}} \leq \frac{w\hat{\sigma}^2_{\text{gauge}}}{\chi^2_{1-\alpha/2,u}}, \tag{3-3}$$

where:

$$w = \left(\hat{\sigma}^2_{\text{gauge}}\right)^2 \left[\frac{(1/np)^2 MS_O^2}{o-1} + \frac{[(pn-1)/pn]^2 MS_R^2}{df_E}\right]^{-1}.$$

Step 6

Repeat simulation Step 4 and Step 5 for 10,000 times. Then choose the mean as the width of confidence interval for variability of measurement.

Based on the above algorithm, one can obtain the widths of confidence interval of measurement variability for any (n, p, k) combination under the same total measurement number through computer simulation. For example, under the same total measurement number 80, Table IV shows that one can get the shortest width of confidence interval of measurement variability if (n, p, k) combination = (10, 4, 2), as indicated by the italicised figures no matter what values of σ_R^2, σ_O^2, and σ_{PO}^2 are. Thus, given the totalmeasurementnumber = 80, the ideal selection of GR&R parameters would be: $n = 10$ samples, $p = 4$ inspectors, and $k = 2$ repeated measurements.

Accordingly, the optimal combination of (n, p, k) for total measurement numbers 40, 60, 90, 120, 160, 180, and 240 can be done in a similar way. If we set as $\sigma_O^2 = 1.0$, $\sigma_{PO}^2 = 0.5$, and $\sigma_R^2 = 1.0$, the results of computer simulation for total measurement numbers 40, 60, 90, 120, 160, 180, and 240 are summarized in Table V.

The following are the major findings of our simulation results:

- if the total measurement number is 40, then the optimal combination of (n, p, k) is taking five samples, assigning four inspectors, and replicating measurement twice;
- if the total measurement number is 60, then the optimal combination of (n, p, k) is taking five samples, assigning four inspectors, and replicating measurement three times;
- if the total measurement number is 90, then the optimal combination of (n, p, k) is taking 15 samples, assigning three inspectors, and replicating measurement twice;
- if the total measurement number is 120, then the optimal combination of (n, p, k) is taking 15 samples, assigning four inspectors, and replicating measurement twice;
- if the total measurement number is 160, then the optimal combination of (n, p, k) is taking 20 samples, assigning four inspectors, and replicating measurement twice;
- if the total measurement number is 180, then the optimal combination of (n, p, k) is taking 15 samples, assigning four inspectors, and replicating measurement three times; and
- if the total measurement number is 240, then the optimal combination of (n, p, k) is taking 20 samples, assigning four inspectors, and replicating measurement three times.

Table V indicates that for total measurement number 120, the two combinations with $p = 4$ have smaller widths of confidence intervals than the rest four combinations, and the two combinations with $p = 3$ also have smaller widths of confidence intervals than two combinations with $p = 2$. Similar findings can also be found with $npk = 160$, 180 and 240 etc. In general, under the same total measurement number, those (n, p, k) combinations with larger number of inspector $p$tend to have smaller widths of confidence intervals. Besides, with same total measurement number and same number of inspectors p, combination with larger sample size of parts n tend to have smaller widths of confidence intervals. For example, with totalmeasurementnumber = 120 and $p = 3$, combination (20, 3, 2) have smaller width of confidence interval than

Table IV. The widths of confidence interval for measurement variability under four different (*n*, *p*, *k*) combination (total measurement number = 80) and various values of source of variation

n	p	k	σ^2_{gauge}								
			$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 0.5$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 1.0$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 1.5$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 0.5$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 1.0$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 1.5$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 0.5$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 1.0$	$\sigma^2_O = 0.5$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 1.5$
5	4	4	3.43	3.31	3.38	4.16	4.04	4.17	4.99	4.94	5.04
10	2	4	31.25	19.39	13.79	22.80	16.09	13.56	22.01	16.74	15.06
10	4	2	*2.98*	*3.00*	*3.22*	*3.22*	*3.39*	*3.57*	*3.63*	*3.82*	*4.04*
20	2	2	24.78	16.41	13.00	18.24	13.18	11.57	14.32	11.56	11.20
			$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 0.5$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 1.0$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 1.5$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 0.5$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 1.0$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 1.5$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 0.5$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 1.0$	$\sigma^2_O = 1.0$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 1.5$
5	4	4	6.99	6.27	5.95	7.17	6.76	6.54	7.94	7.51	7.29
10	2	4	124.24	75.80	52.93	94.57	59.30	44.55	77.68	54.95	41.22
10	4	2	*6.22*	*5.75*	*5.74*	*6.13*	*5.85*	*5.87*	*6.34*	*6.09*	*6.17*
20	2	2	115.41	75.44	50.66	75.07	50.67	41.71	50.68	39.91	31.86
			$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 0.5$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 1.0$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 1.5$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 0.5$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 1.0$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 1.0$ $\sigma^2_R = 1.5$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 0.5$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 1.0$	$\sigma^2_O = 1.5$ $\sigma^2_{PO} = 1.5$ $\sigma^2_R = 1.5$
5	4	4	11.02	9.78	9.26	10.95	10.23	9.59	11.32	10.69	10.17
10	2	4	265.35	189.29	130.52	196.28	132.35	96.74	164.19	119.54	87.34
10	4	2	*10.51*	*9.44*	*9.03*	*9.68*	*9.09*	*8.67*	*9.49*	*8.92*	*8.82*
20	2	2	251.44	176.35	117.77	166.13	121.00	95.72	128.22	93.63	77.01

Table V.
The width of confidence interval for measurement variability under different (n, p, k) combination and various total measurement number

n	p	k	σ^2_{gauge} $\sigma^2_O = 1.0$ $\sigma^2_{PO} = 0.5$ $\sigma^2_R = 1.0$
Total measurement = 40			
5	2	4	105.56
5	4	2	*7.28*
10	2	2	95.18
Total measurement = 60			
5	3	4	11.25
5	4	3	*6.53*
10	2	3	84.38
10	3	2	10.77
15	2	2	80.08
Total measurement = 90			
10	3	3	9.57
15	3	2	*9.44*
15	2	3	70.05
Total measurement = 120			
10	3	4	9.28
10	4	3	5.47
15	2	4	67.73
15	4	2	*5.42*
20	2	3	65.98
20	3	2	9.00
Total measurement = 160			
10	4	4	5.40
20	2	4	60.90
20	4	2	*5.14*
Total measurement = 180			
15	3	4	8.73
15	4	3	*5.23*
20	3	3	8.54
Total measurement = 240			
15	4	4	5.11
20	3	4	8.17
20	4	3	*5.08*

combination (10, 3, 4). Thus, with a fixed total measurement number and fixed number of inspectors p, a larger sample size n leads to a smaller width of confidence interval.

When performing a GR&R study, it is recommended that the total measurement number be decided based on the consideration of acceptable cost, and it is also preferred to assign more inspectors to perform measurement. However, if the number of inspectors is fixed, then a combination with larger sample size and less repeated measurements is desired.

4. Numerical example

One TFT-LCD manufacturer located in Taiwan produces high-resolution microscopes. Among various manufacturing processes, the sealing process, of which sealing gum applied to two glasses, is a critical one. If more gum applied to the glasses, it will cause residual/splatter problem. On the other hand, the two glasses cannot be properly sealed if less gum is applied. Therefore, it is necessary to perform statistical process control (SPC) and the variability of measurement/GR&R has to be analyzed prior to the SPC study, otherwise the results of SPC will be greatly influenced. The following procedures are suggested to perform GR&R study for practitioners.

Step 1
Decide the product's quality characteristic, gauge for measurement, and specifications of quality characteristic.

In this case, the product's quality characteristic is the absorbing amount of gum for microscopes and the specifications of quality characteristic is 0.1 ~ 0.6 mm.

Step 2
Decide the optimal allocation of measurement parameters (sample size, number of inspectors, replicate measurements).

The following allocation of measurement parameters is currently used by the TFT-LCD manufacturer:

- sample size (n): 10;
- number of inspectors (p): 3; and
- replicate measurements (k): 4.

According to the figures shown in Table V, the width of confidence interval for measurement variability will be equal to 9.28 under the totalmeasurementnumber = 120. However, it is recommended that the optimal allocation of measurement parameters for performing GR&R study be either $n = 15$, $p = 4$, $k = 2$ or $n = 10$, $p = 4$ and $k = 3$. Then the width of confidence interval for measurement variability can be reduced from 9.28 to 5.42 or 5.47 for the above (15, 4,2) and (10, 3, 4) allocation respectively.

Step 3
Perform actual measurement and collect data.

After deciding the optimal allocation of measurement parameters, practitioners randomly select samples and assign inspectors to measure each sample.

Step 4
Estimate the GR&R of measuring process.

ANOVA, Classical GR&R, and Long Form methods can be used to estimate P/T ratio, i.e. the ratio of gauge repeatability and reproducibility to the specification width.

Step 5
Evaluate the adequacy of measurement system based on the above P/T ratio.

5. Conclusion

An adequate measurement system plays very important role in quality improvement in industry. This paper discusses the suitability of (n, p, k) selection using the current empirical method versus a statistical method by picking up the (n, p, k) combination with the shortest confidence interval. Based on our simulation results, some recommendations are proposed for selecting the optimal combinations of sample size, number of inspectors, and replicate measurements:

- It is suggested that the total measurement number be decided prior to performing the GR&R study, then one can select the optimal combination of sample size, number of inspectors, and repeated measurements according to Tables IV and V.
- With the same total measurement number, quality practitioners should assign more inspectors. However, if number of inspectors is fixed, then a combination with more samples and less repeated measurements is suggested.
- Since the (n, p, k) combination with the shortest confidence interval is used as a criterion for optimal allocation of GRR parameters and ANOVA method includes the variation of interaction between product and inspector, it is also suggested that quality practitioners use the ANOVA method to perform the GR&R study.

References

Automotive Industry Action Group (AIAG) (1995), "Production part approval process", *AIAG Reference Manual*, Southfield, MI.

Automotive Industry Action Group (AIAG) (1997), "Measurement systems analysis", *AIAG Reference Manual*, Southfield, MI.

Burdick, R.K. and Larsen, G.A. (1997), "Confidence intervals on measures of variability in R&R studies", *Journal of Quality Technology*, Vol. 29 No. 3, pp. 261-73.

DataMyte Editing Group (1989), *DataMyte Handbook*, 4th ed., DataMyte Corporation, Painesville, OH, pp. 17-25.

Montgomery, D.C. and Runger, G.C. (1993a), "Gauge capability analysis and designed experiments. Part I: basic methods", *Quality Engineering*, Vol. 6 No. 1, pp. 115-35.

Montgomery, D.C. and Runger, G.C. (1993b), "Gauge capability analysis and designed experiments. Part II: experimental design models and variance component estimation", *Quality Engineering*, Vol. 6 No. 2, pp. 289-305.

Pan, J.N. and Lee, W.R. (2003), *Quality Management*, Hwa-Tai Book Company, Taipei.

Tsai, P. (1989), "Variable gauge repeatability and reproducibility study using the analysis of variance method", *Quality Engineering*, Vol. 1 No. 1, pp. 107-15.

Further reading

Hogg, R.V. and Craig, A.T. (1995), *Introduction to Mathematical Statistics*, Prentice-Hall, Englewood Cliffs, NJ.

Juran, J.M. (1974), *Quality Control Handbook*, 3rd ed., McGraw-Hill, New York, NY.

Kane, V.E. (1986), "Process capability indices", *Journal of Quality Technology*, Vol. 18 No. 1, pp. 41-52.

Majeske, K.D. and Andrews, R.W. (2002), "Evaluating measurement systems and manufacturing processes using three quality measures", *Quality Engineering*, Vol. 15 No. 2, pp. 243-51.

Mandel, J. (1972), "Repeatability and reproducibility", *Journal of Quality Technology*, Vol. 4 No. 2, pp. 74-85.