Measurement System Assessment Via Experimental Study

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目 錄

第一章 研究動機

一個好的量測系統，對於成功的品質改善以及統計制程規劃而言，是相當重要的，因為不好的量測系統會導致事半功倍，可能產生品質不好的產品以及不好的製程。所以在此針對量測變異(Gauge Repeatability and Reproducibility) 做探討。

第二章 量測變異之介紹

量測儀器(Gauge)可能產生一種或多種的變異型態，如：Accuracy, Linearity, Stability, Reproducibility, Repeatability. 而在此只針對 Gauge Repeatability and Reproducibility 的條件來做探討，而其定義如下：

- Repeatability (Gauge Variation):

  The variation in measurement observed when one operator repeatedly measures the same characteristic in the same place on the same part.

- Reproducibility (Operator Variation):

  The variation in average measurements obtained by several operators while repeatedly measuring the same characteristic in the place on the same group of parts.

這兩條件可以簡稱為 R&R，所以 Gauge Repeatability and Reproducibility 可以簡稱為 GR&R。

一個產品的變異，包含其本身製造時所產生的變異，以及量測時所帶來的變異，其描述如下之等式：

$$\sigma_T^2 = \sigma_p^2 + \sigma_m^2$$

σ_T^2 = Total variance

σ_p^2 = process variance

σ_m^2 = measurement variance

在此我們感興趣的是量測時所帶來的變異，而量測變異也包含了量測儀器本身的變異以及量測員所來的變異，其描述如下之等式：

$$\sigma_m^2 = \sigma_e^2 + \sigma_o^2$$

σ_m^2 = measurement variance

σ_e^2 = gauge variance

σ_o^2 = operator variance

第三章　量測變異計算方法

在此將介紹 Long form method, 5.15 sigma method 以及 ANOVA method 來計算 GR&R。

3.1 Long form method

利用下表，在空格中依序填入欲填的數值，即可求出 GR&R 的數值。

表 3.1 Long form

	1	2	3	4	5	6	7	8	9	10	11	12
Operator	A ·				B ·				C ·			
Sample #	1st Trial	2nd Trial	3rd Trial	Range	1st Trial	2nd Trial	3rd Trial	Range	1st Trial	2nd Trial	3rd Trial	Range
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
Totals												
				$\bar{R}_A$				$\bar{R}_B$				$\bar{R}_C$
		Sum				Sum				Sum		
		$\bar{\bar{X}}_A$				$\bar{\bar{X}}_B$				$\bar{\bar{X}}_C$		

$\bar{R}_A$	
$\bar{R}_B$	
$\bar{R}_C$	
Sum	
$\bar{\bar{R}}$	

# of Trials	D_4
2	3.27
3	2.58

($\bar{\bar{R}}$) x (D_4) = UCL_R *

(　　) x (　　) =

Max. $\bar{\bar{X}}$	
Min. $\bar{\bar{X}}$	
$\bar{\bar{X}}$ Diff.	

*Limit of individual R's. Circle those that are beyond this limit. Identify the cause and correct. Repeat these readings using the same appraiser and unit as originally used or discard values and reaverage and compute R and the limiting value, UCL_R from the remaining observations.

Notes: _____

Figure 1. Gauge repeatability and reproducibility data sheet (long method).

3.3 ANOVA method

在利用 ANOVA method 之前，必須先假設四個基本假設：

1. The operator, part, interaction, and gauge effects are additive.

2. The operator, part and gauge effects are normally distributed with zero mean and some variance.

3. The gauge errors must be independent of the operator, part, and interaction effects.

4. Each observation must have a common population variance.

由下表的 ANOVA Table 可算出 Empected Mean Squares。

TABLE 3. ANOVA Table for a Gauge R & R Study

Source	Degrees of Freedom	Sum of Squares	Expected Mean Squares
part	$p-1$	$S_P = \sum_i on(\bar{y}_i - \bar{y})^2$	$\theta_0 = on\sigma^2_P + n\sigma^2_{OP} + \sigma^2_E$
operator	$o-1$	$S_O = pn\sum_j (\bar{y}_j - \bar{y})^2$	$\theta_1 = pn\sigma^2_O + n\sigma^2_{OP} + \sigma^2_E$
operator x part	$(o-1)(p-1)$	$S_{OP} = n\sum_i \sum_j (\bar{y}_{ij} - \bar{y}_i - \bar{y}_j + \bar{y})^2$	$\theta_2 = n\sigma^2_{OP} + \sigma^2_E$
error (within O,P)	$op(n-1)$	$S_E = \sum_i \sum_j \sum_k (y_{ijk} - \bar{y}_{ij})^2$	$\theta_3 = \sigma^2_E$
total(corrected)	$opn-1$	$\sum_i \sum_j \sum_k (y_{ijk} - \bar{y})^2$	

Part No. & Name _____ Gage Name _____ Date _____

Characteristic _____ Gage No. _____ Performed by _____

Specification _____ Gage Type _____

From Data Sheet: $\bar{\bar{R}}$ [ ] From Data Sheet: $\bar{X}$ Diff. [ ]

Trials	2	3
K_1	4.56	3.05

MEASUREMENT UNIT ANALYSIS

Repeatability - Equipment Variation (E.V.)

E.V. = ($\bar{\bar{R}}$) x (K_1)

[ ] = ( ) x ( )

% TOLERANCE ANALYSIS

%E.V. = (E.V. ÷ Tolerance) x 100

= [( ) ÷ ( )] x 100

[ ] % =

Operators	2	3
K_2	3.65	2.70

Reproducibility - Appraiser Variation (A.V.)

A.V. = ($\bar{X}_{Diff.}$) x (K_2)

[ ] = ( ) x ( )

%A.V. = (A.V. ÷ Tolerance) x 100

= [( ) ÷ ( )] x 100

[ ] % =

NOTE: All calculations are based upon predicting 99% of the area under normal curve.

* A negative value under the square root sign causes the appraiser variation to the default to zero.

Repeatability & Reproducibility (R & R)

R & R = $\sqrt{(E.V.)^2 + (A.V.)^2}$

[ ] = $\sqrt{(\quad)^2 + (\quad)^2}$

%R & R = $\sqrt{(\%E.V.)^2 + (\%A.V.)^2}$

[ ] % = $\sqrt{(\quad)^2 + (\quad)^2}$

Figure 2. Gauge repeatability and reproducibility report.

3.2 5.15 sigma method

利用以下的公式，亦可求出 GR&R 的數值。

$$\sigma_e = \frac{\bar{\bar{R}}}{d_2}$$

$$\sigma_o = \frac{\overline{X}_{Diff}}{d_2}$$

$$\frac{GRR}{Tolerance} = \frac{5.15\sqrt{\sigma_e^2 + \sigma_o^2}}{Tolerance}$$

由 ANOVA Table 可推算出，當 $Ho:\sigma^2_{op}=0$ is rejected 時，其各種變異被估算如下：

$$\begin{aligned}\widehat{\sigma}^2_O &= (MS_O - MS_{OP})/pn\\ \widehat{\sigma}^2_P &= (MS_P - MS_{OP})/on\\ \widehat{\sigma}^2_{OP} &= (MS_{OP} - MS_E)/n\\ \widehat{\sigma}^2_E &= MS_E.\end{aligned}$$

而當 $Ho:\sigma^2_{op}=0$ is not reject 時，其各種變異被估算如下：

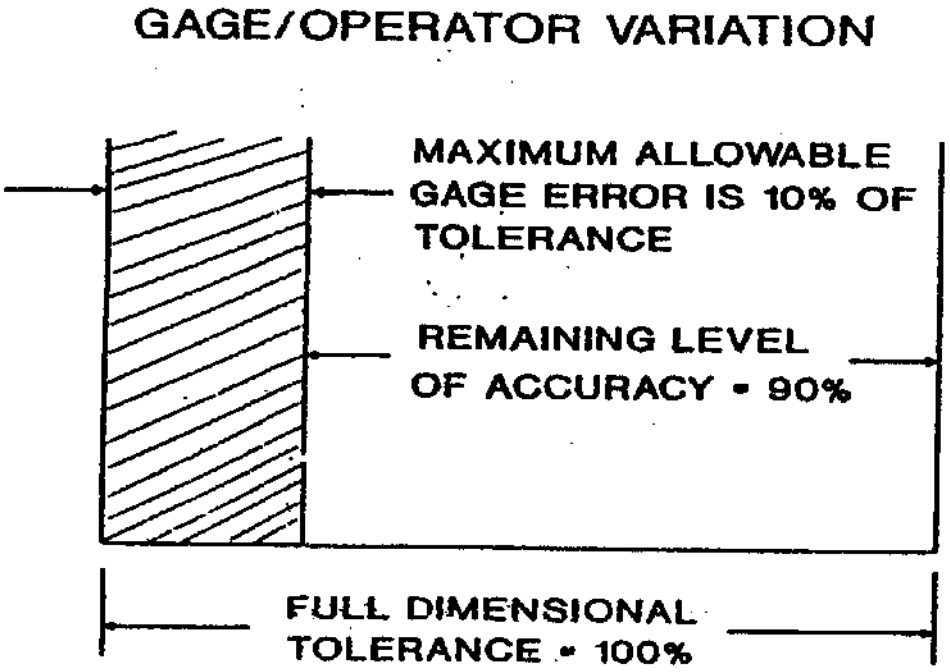
$$\begin{aligned}\widehat{\sigma}^2_O &= (MS_O - MS_{E'})/pn\\ \widehat{\sigma}^2_P &= (MS_P - MS_{E'})/on\\ \widehat{\sigma}^2_E &= MS_{E'} = (S_E + S_{OP})/(opn - o - p + 1)\end{aligned}$$

其交互作用項的變異納入了誤差項變異中。

將所求算出的變異數，求其標準差，再乘上 5.15，如同下表作法，即可求出 GR&R 的值。

Variance	Standare Deviation	5.15 S.D.	Percent of Tolerance
Repeatability: $\sigma_e^2 =$			
Reproducibility: $\sigma_o^2 =$			
Repeatability and Reproducibility: $\sigma_e^2 + \sigma_o^2 =$		GR&R	$\frac{GR\&R}{Tolerance}$

3.4 GR&R 決策準則



$$\frac{GR\&R}{Tolerance} < 10\% \rightarrow \text{Good}$$

$$10\% < \frac{GR\&R}{Tolerance} < 25\% \rightarrow \text{Acceptable}$$

$$\frac{GR\&R}{Tolerance} > 25\% \rightarrow \text{Bad}$$

由上之檢測力(P/T 值)可知：當 P/T 值小於 10 %時，其量測儀器不錯，可繼續使用；如果 P/T 值介於 10 %～25 %，量測儀器使用與否可視各公司決定；如果 P/T 值大於 25 % ，表示此量測儀器很差，可考慮換掉。

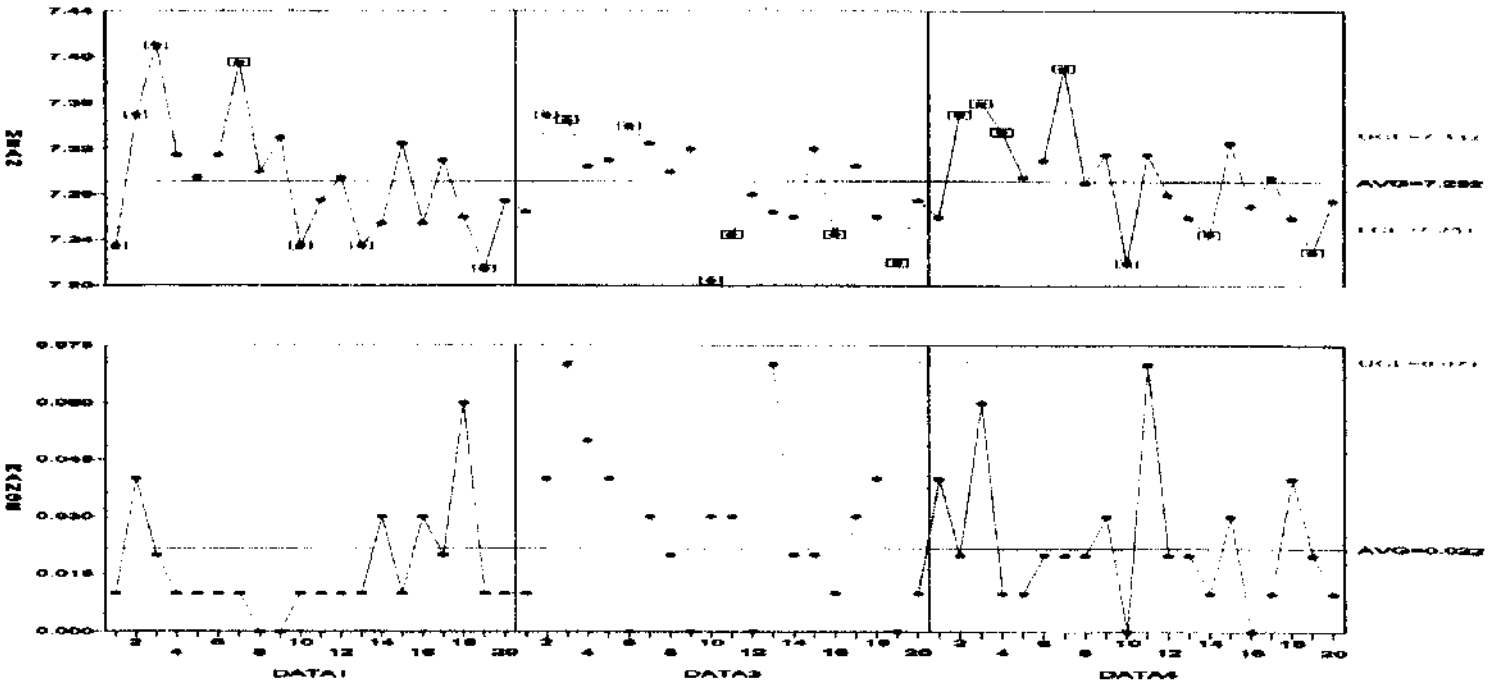
第四章　實驗數值分析

隨機尋求三位量測員，量測鉛筆之寬度，每位量測員各自量 20 支，重覆量兩次，使用量測儀器為游標卡尺以及微量器，其規格寬度定為：

$$7.3 \pm 0.2 \text{ mm}$$

4.1 游標卡尺之數值分析

在計算 GR&R 值之前必須先檢查製程是否為穩定(in control)，由下圖的 R-chart 可看出製程都是 in control。



Long form method

$$\text{Repeatability} = k_1 \times \bar{\bar{R}} = 4.56 \times 0.0217 = 0.099$$

($k_1 = 4.56$ for 2 trials, 3.05 for 3 trials)

$$\text{Reproducibilty} = \sqrt{(k_2 \bar{X}_{Diff})^2 - \frac{\text{Repeatability}^2}{n \times r}}$$

$$= \sqrt{(2.7 \times 0.007)^2 - (\frac{0.099^2}{20 \times 2})} = 0.011$$

(k2 = 3.65 for 2 operators, 2.70 for 3 operators)

$$\therefore \text{GRR} = \sqrt{\text{Repeatability}^2 + \text{Reproducibility}^2} = 0.0996$$

$$\therefore \frac{\text{GRR}}{\text{Tolerance}} = \frac{0.0996}{0.4} = 0.249 = 24.9\%$$

5.15 sigma method

$$\sigma_e = \frac{\bar{\bar{R}}}{d_2} = \frac{0.0217}{1.128} = 0.019$$

$$\sigma_o = \frac{\bar{X}_{Diff}}{d_2} = 0.007 \times 0.524 = 0.0037$$

$$\frac{GR\&R}{Tolerance} = \frac{5.15\sqrt{0.019^2 + 0.0037^2}}{0.4} = 0.2492 = 24.92\%$$

ANOVA method

由下表 ANOVA Table 可發現其交互作用項並不顯著，

所以在此可考慮刪除，納入誤差項中。

Source	DF	SS	MS	F	P
OPERATOR	2	0.0000633	0.0000317	0.12	0.886
PART	9	0.0552270	0.0061363	23.60	0.000
OPERATOR*PART	18	0.0046033	0.0002557	0.98	0.501
Error	30	0.0078001	0.0002600		
Total	59	0.0676937			

由下表的 ANOVA Table 可算出 Empected Mean Squares。

Source	DF	SS	MS	F	P
OPERATOR	2	0.0000633	0.0000317	0.12	0.885
PART	9	0.0552270	0.0061363	23.75	0.000
Error	48	0.0124034	0.0002584		
Total	59	0.0676937			

將所求算出的變異數，填入下表，求其 GR&R 之值。

Variance	Standare Deviation	5.15 S.D.	Percent of Tolerance
Repeatability: $\sigma_e^2 = 0.000466$	0.021587	0.111173	27.79 %
Reproducibility: $\sigma_o^2 = 0.000004675$	0.0021622	0.011135	2.78 %
Repeatability and Reproducibility: $\sigma_e^2 + \sigma_o^2 =$ 0.0004707	0.02170	GR&R= 0.111755	27.94 %

將以上三種方法計算 GR&R 之值，列表如下：可發現 long form method 求出 GR&R 的值和 5.15 sigma method 很接近，所以在業界通常會使用 5.15 sigma method 來計算，因為計算方法比 long form method 要來的簡單。另外由 ANOVA method 所求出的值也只比其它兩種方法所求出的值大一點。

由下表也可發現：GR&R 中，其重複性(Repeatability)占的比例較再現性(Reproducibility)要來的大，所以在我們這次實驗結果可得知，在量測變異中，其大部份的變異是由量測儀器本身所帶來的變異。

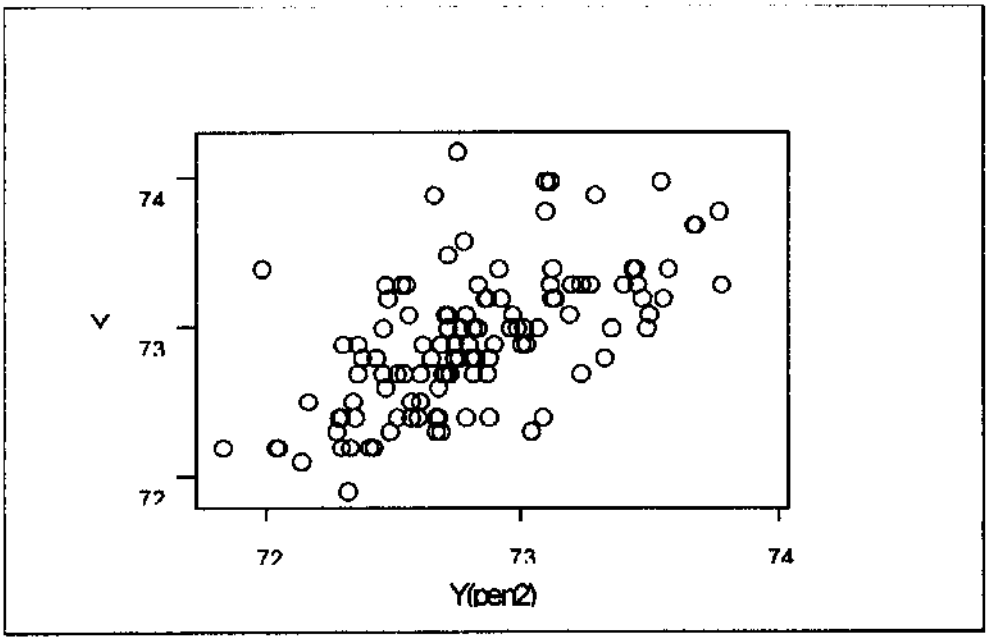
游標卡尺

Method	*GRR* / *Tolerance*	Re *peatability* / *Tolerance*	Re *producibility* / *Tolerance*
Long form method	24.9 %	24.75 %	2.75 %
5.15 sigma method	24.92 %	24.46 %	4.76 %
Anova Method	27.94 %	27.79 %	2.78 %

4.2 游標卡尺與微量器之相關檢定

此實驗使用了游標卡尺以及微量器，在此檢定兩者間之相關性，檢測兩者有何不同。

由 pearson correlations 可求出相關值為 0.612，因為游標卡尺以及微量器皆屬於精密儀器，所以相關值 0.612 的相關度算蠻高的。下圖為游標卡尺以及微量器之相關圖，兩者是呈現正相關。



遠；另外，微量器所量之值

第五章　未來研究方向

1. 量測變異(GR&R)對計量管制圖之影響分析。

2. 量測變異對製程能力指標之影響分析。

3. 量測變異決策準則之探討。

參考文獻

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2. Michael Hamada and Sam Weerahandi (2000). "Measurement System Assessment Via Generalized Inference". *Journal of Quality Technology* 32,pp241-253.

2. Duane A. Floyd and Carl J. Laurent (1995-96). "Gauging : An Underestimated Consideration In The Application Of Statistical Process Control". *Quality Engineering* 8(1),pp13-29.

4. "Implementing Six Sigma", Forrest W. Breyfogle III.

量測重複性與再現性之實例

資料放至

D: > user > pan >量測資料> pen1(游標卡尺)

D: > user > pan >量測資料> pen2(微量器)

說明:

資料 pen1：隨機尋求三位量測員，量測鉛筆之寬度，每位量測員各自量 10 支鉛筆，重複量 2 次，使用**游標卡尺**。

資料 pen2：隨機尋求三位量測員，量測鉛筆之寬度，每位量測員各自量 10 支鉛筆，重複量 2 次，使用**微量器**。

軟體操作:

使用 SPCII 軟體

1. 在 Chart 中選擇 Gauge R&R
2. 在 Gauge R&R 畫面中選

   appraisers: 量測員 1 – 量測員 3

   Trials per part: 選 2

   Control Chart: 選 repeatability

   Repeatability: 選 combined

   其餘不變(如要 statistics,要記得勾選)

樣本	重複次數	量測員1	量測員2	量測員3	DATA4	COMMENTS
1	1	7.24	7.27	7.24		
1	2	7.23	7.26	7.28		
2	1	7.33	7.33	7.36		
2	2	7.37	7.37	7.34		
3	1	7.40	7.31	7.33		
3	2	7.42	7.38	7.39		
4	1	7.31	7.33	7.34		
4	2	7.32	7.28	7.33		
5	1	7.29	7.33	7.30		
5	2	7.30	7.29	7.29		
6	1	7.32	7.34	7.32		
6	2	7.31	7.34	7.30		
7	1	7.40	7.31	7.38		
7	2	7.39	7.37	7.40		
8	1	7.30	7.31	7.28		
8	2	7.30	7.29	7.30		
9	1	7.33	7.32	7.33		
9	2	7.33	7.32	7.30		
10	1	7.24	7.19	7.22		
10	2	7.23	7.22	7.22		

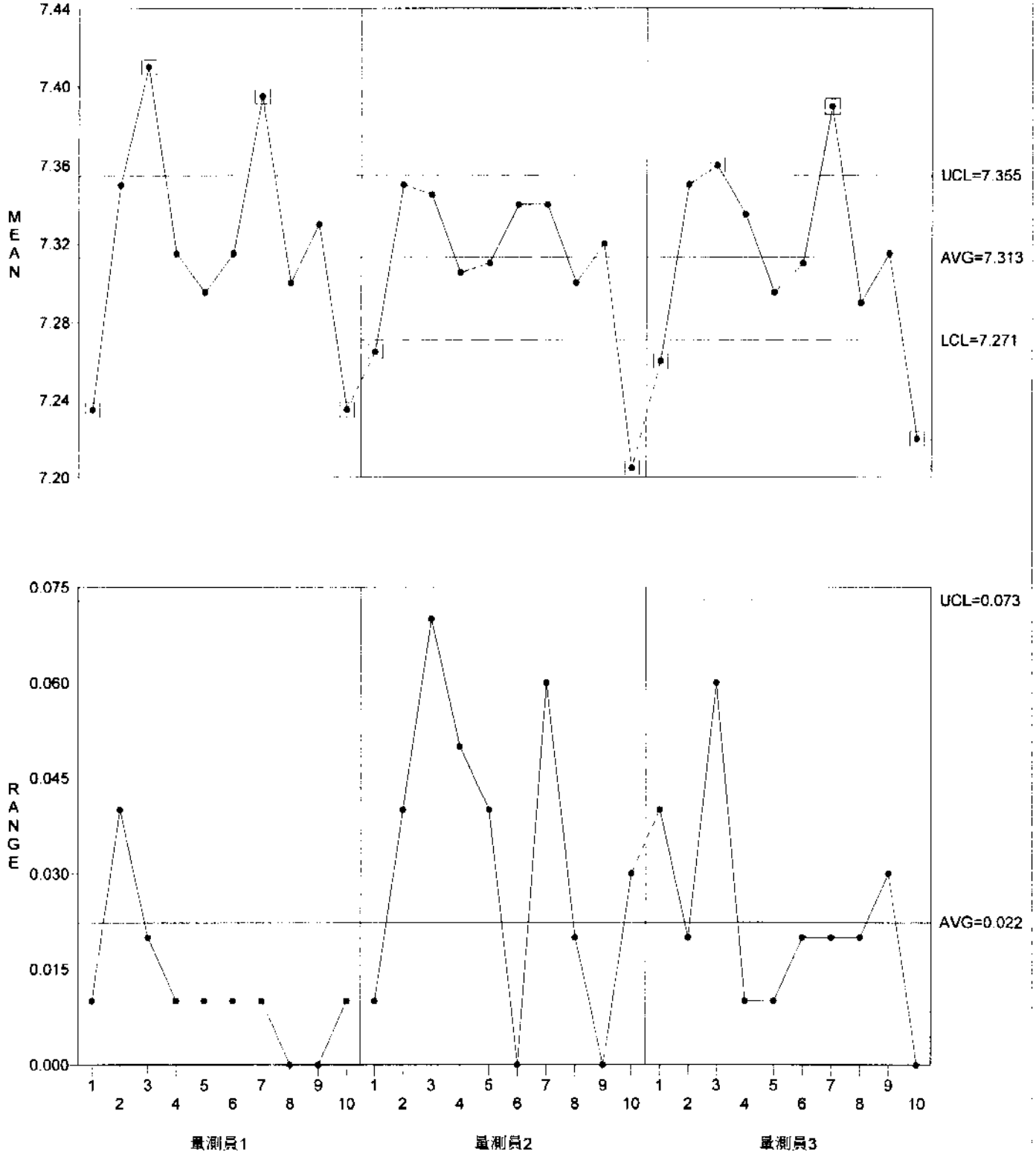

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CHART3: Gage R and R: pen1(游標卡尺)

STATISTICS FOR REPEATABILITY AND REPRODUCIBILITY

Appraisers:	量測員1, 量測員2, 量測員3
Parts Selected:	10
Trials Per Part:	2
Sigma Level:	2.575
Process Sigma (calculated):	0.053
Discrimination Ratio:	3.4

Measurement Units	% Total Variation
Repeatability Equipment Variation (EV)	
EV = 0.102	%EV = 37.572
Reproducibility Appraiser Variation (AV)	
AV = 0.014	%AV = 5.351
Repeatability & Reproducibility (R&R)	
R&R = 0.103	%R&R 37.951
Part Variation (PV)	
PV = 0.251	%PV = 92.519
Total Variation (TV)	
TV = 0.271	

AVERAGE OF MEASUREMENTS:

PART	量測員1	量測員2	量測員3	AVERAGE	RANGE
1	7.2350	7.2650	7.2600	7.2533	0.0300
2	7.3500	7.3500	7.3500	7.3500	0.0000
3	7.4100	7.3450	7.3600	7.3717	0.0650
4	7.3150	7.3050	7.3350	7.3183	0.0300
5	7.2950	7.3100	7.2950	7.3000	0.0150
6	7.3150	7.3400	7.3100	7.3217	0.0300
7	7.3950	7.3400	7.3900	7.3750	0.0550
8	7.3000	7.3000	7.2900	7.2967	0.0100
9	7.3300	7.3200	7.3150	7.3217	0.0150
10	7.2350	7.2050	7.2200	7.2200	0.0300
AVERAGE	7.3180	7.3080	7.3125	7.3128	0.0280

RANGE OF MEASUREMENTS:

PART	量測員1	量測員2	量測員3	
1	0.0100	0.0100	0.0400	
2	0.0400	0.0400	0.0200	
3	0.0200	0.0700	0.0600	
4	0.0100	0.0500	0.0100	
5	0.0100	0.0400	0.0100	
6	0.0100	0.0000	0.0200	
7	0.0100	0.0600	0.0200	
8	0.0000	0.0200	0.0200	
9	0.0000	0.0000	0.0300	
10	0.0100	0.0300	0.0000	COMBINED
AVERAGE:	0.0120	0.0320	0.0230	0.0223
STD DEV:	0.0103	0.0276	0.0198	0.0198
REPEAT:	0.0533	0.1421	0.1021	0.1018

樣本	重複次數	量測員1	量測員2	量測員3	DATA4	COMMENTS
1	1	7.266	7.253	7.287		
1	2	7.268	7.246	7.242		
2	1	7.378	7.339	7.269		
2	2	7.368	7.367	7.277		
3	1	7.354	7.296	7.345		
3	2	7.274	7.378	7.265		
4	1	7.318	7.252	7.291		
4	2	7.285	7.340	7.311		
5	1	7.302	7.254	7.271		
5	2	7.306	7.261	7.273		
6	1	7.355	7.344	7.347		
6	2	7.349	7.312	7.348		
7	1	7.311	7.271	7.309		
7	2	7.328	7.343	7.309		
8	1	7.301	7.255	7.282		
8	2	7.295	7.268	7.283		
9	1	7.326	7.286	7.283		
9	2	7.323	7.312	7.335		
10	1	7.228	7.231	7.229		
10	2	7.227	7.182	7.240		

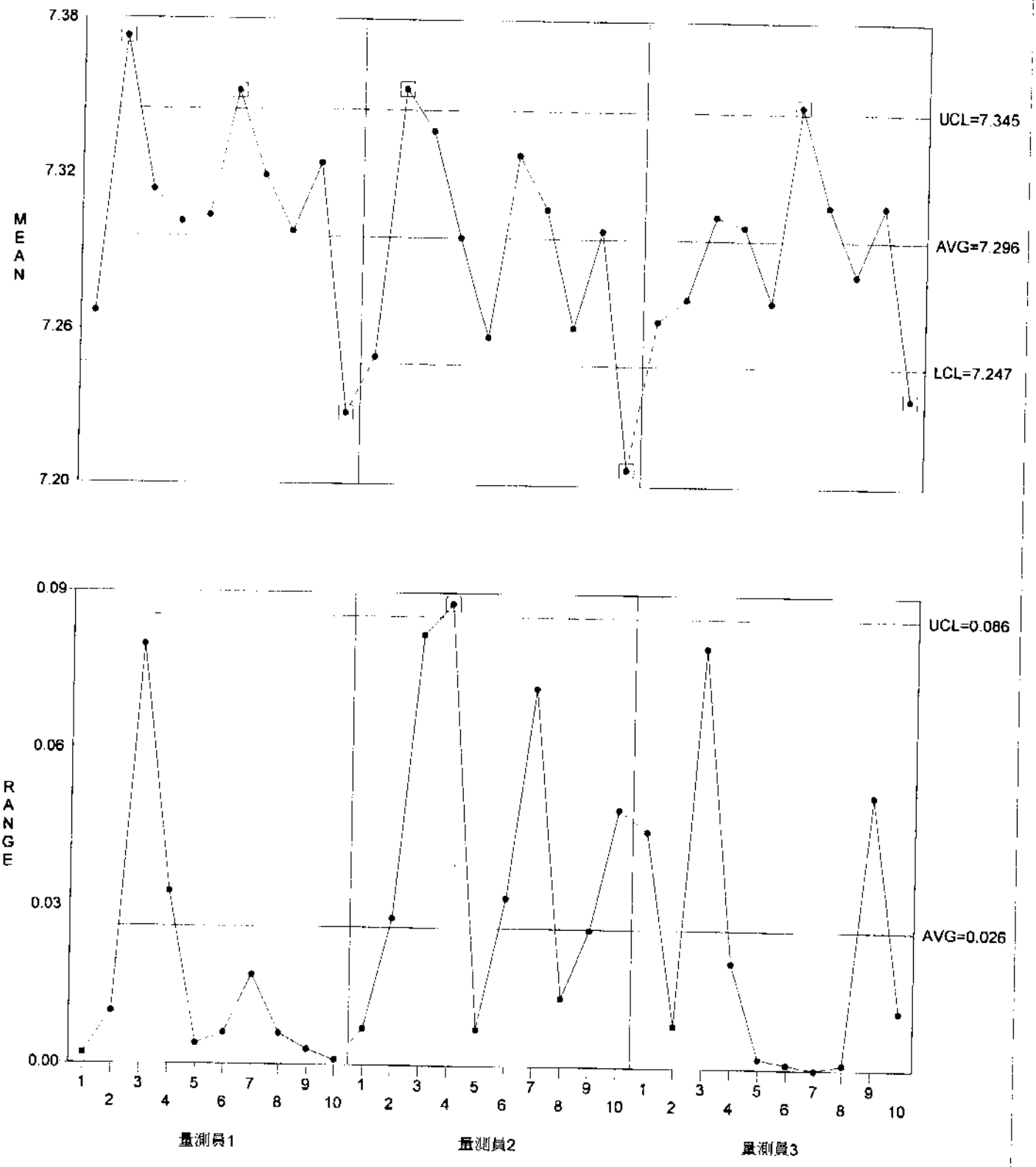

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CHART2: Gage R and R: pen2(微量器)

STATISTICS FOR REPEATABILITY AND REPRODUCIBILITY

Appraisers:	量測員1, 量測員2, 量測員3
Parts Selected:	10
Trials Per Part:	2
Sigma Level:	2.575
Process Sigma (calculated):	0.045
Discrimination Ratio:	2.2

Measurement Units	% Total Variation
Repeatability - Equipment Variation (EV)	
EV = 0.119	%EV = 51.620
Reproducibility- Appraiser Variation (AV)	
AV = 0.043	%AV = 18.373
Repeatability & Reproducibility (R&R)	
R&R 0.127	%R&R = 54.793
Part Variation (PV)	
PV 0.194	%PV = 83.653
Total Variation (TV)	
TV 0.231	

AVERAGE OF MEASUREMENTS:

PART	量測員1	量測員2	量測員3	AVERAGE	RANGE
1	7.2670	7.2495	7.2645	7.2603	0.0175
2	7.3730	7.3530	7.2730	7.3330	0.1000
3	7.3140	7.3370	7.3050	7.3187	0.0320
4	7.3015	7.2960	7.3010	7.2995	0.0055
5	7.3040	7.2575	7.2720	7.2778	0.0465
6	7.3520	7.3280	7.3475	7.3425	0.0240
7	7.3195	7.3070	7.3090	7.3118	0.0125
8	7.2980	7.2615	7.2825	7.2807	0.0365
9	7.3245	7.2990	7.3090	7.3108	0.0255
10	7.2275	7.2065	7.2345	7.2228	0.0280
AVERAGE	7.3081	7.2895	7.2898	7.2958	0.0328

RANGE OF MEASUREMENTS:

PART	量測員1	量測員2	量測員3	
1	0.0020	0.0070	0.0450	
2	0.0100	0.0280	0.0080	
3	0.0800	0.0820	0.0800	
4	0.0330	0.0880	0.0200	
5	0.0040	0.0070	0.0020	
6	0.0060	0.0320	0.0010	
7	0.0170	0.0720	0.0000	
8	0.0060	0.0130	0.0010	
9	0.0030	0.0260	0.0520	
10	0.0010	0.0490	0.0110	
AVERAGE:	0.0162	0.0404	0.0220	0.0262
STD DEV:	0.0140	0.0348	0.0190	0.0232
REPEAT:	0.0719	0.1793	0.0977	0.1194

COMBINED