設計品質講義
潘浙楠老師

1. 全因子設計
2. 全因子設計範例
3. 部分因子設計
4. 部分因子設計範例
5. 田口方法
6. 田口方法範例
7. 實驗之選擇與其他

- 品質研究的四個階段

1. 下游: " Customer's Quality " , such as fuel consumption, noise, failure rates, pollution, etc.
2. 中游: " Manufacturing Quality " (Spec. + Drawings) Important for production or trading
3. 上游: " Quality of Design" (Robustness of Objective Function) Good for Design & Development after product planning
4. 源流: " Quality of Technology " (Robustness of Technology) Good for Technology Development ever before product planning ; Functionality of Generic Function.

   如: $Y = \beta \bullet M$ Hook's Law

- 品質工程: 提供在設計產品及設計生產過程時對預測在市場會發生故障之技術/方法.

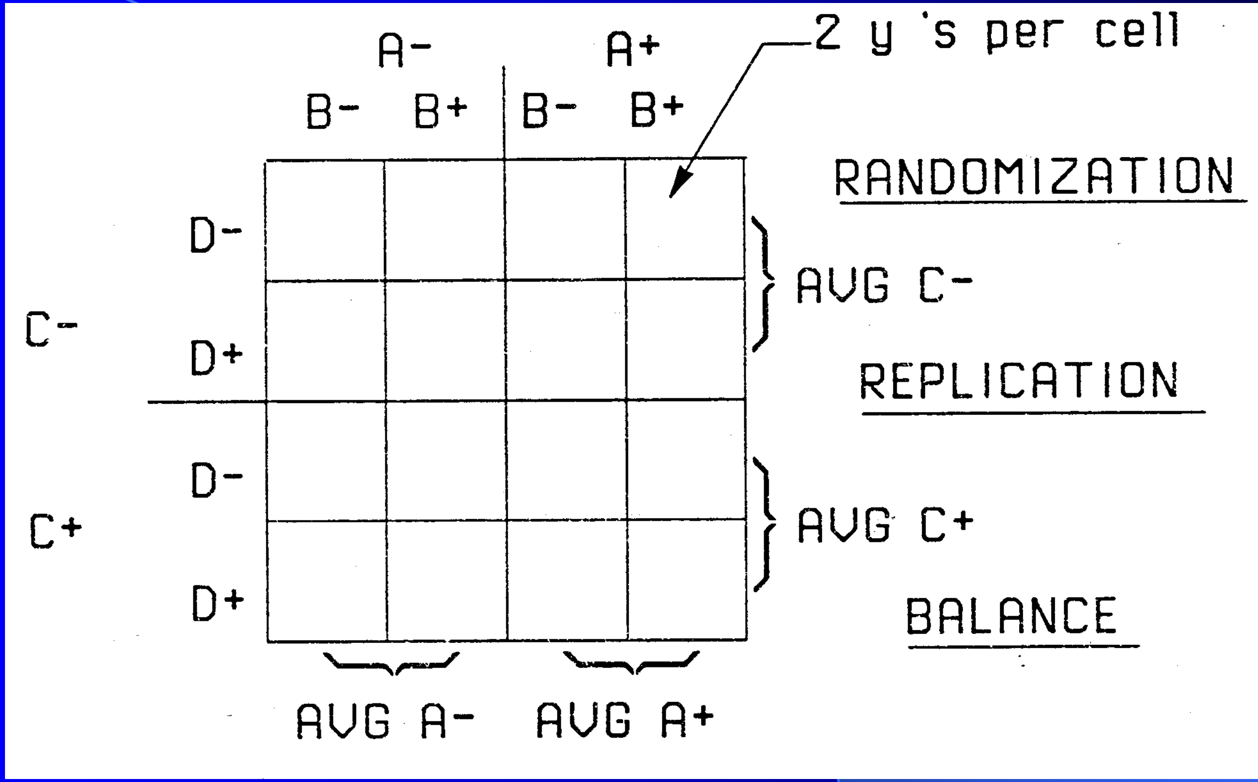
❑ 技術開發之特徵:

1. 先行性 (Technology Readiness): 產品計畫前作技術開發;
設計完後僅須作適當調整.

2. 汎用性 (Flexibility): 不對個別產品作品質改善.
一系列或下一代.

3. 再現性 (Reproducibility): R & D ⇒ Manufacturing ⇒ Market

FULL FACTORIAL
DESIGNED EXPERIMENT (實驗設計之三大原則)

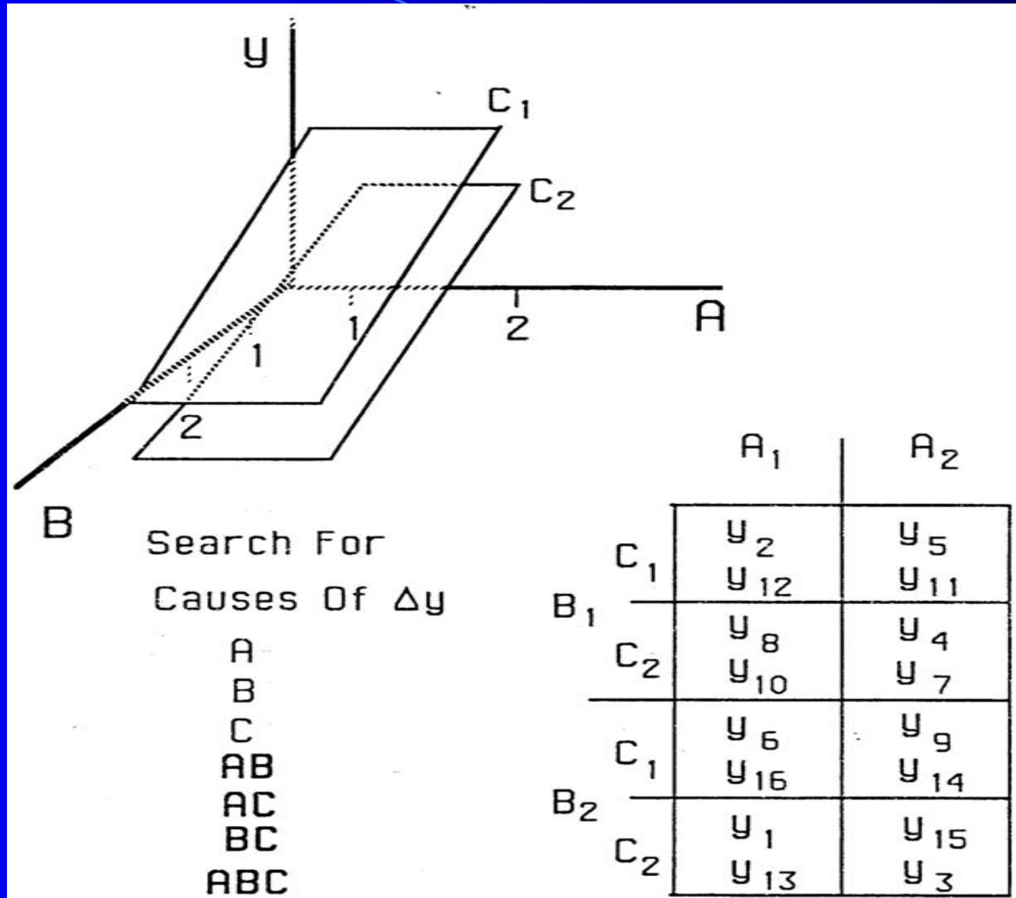


RANDOM NUMBER TABLE

71	85	71	59	57	25	16	30	18	89
92	78	42	63	40	65	25	10	76	29
04	92	17	37	01	36	81	54	63	25
45	19	72	53	32	64	39	71	16	92
15	19	11	87	82	04	51	52	56	24
01	29	14	13	49	83	76	16	08	73
38	38	47	47	61	14	38	70	63	45
66	16	44	94	31	51	32	19	22	46
54	15	58	34	36	72	47	20	00	08
72	84	81	18	34	05	46	65	53	06
18	61	91	36	74	39	52	87	24	84
74	62	77	37	07	81	61	61	87	11
32	39	21	97	63	07	58	61	61	20
78	46	42	25	01	90	76	70	42	35
62	09	53	67	87	40	18	82	81	93
12	30	28	07	83	34	41	48	21	57
76	37	84	16	05	63	43	97	53	63
05	04	14	98	07	67	04	90	90	70
46	97	83	54	82	79	49	50	41	46
47	66	56	43	82	91	70	43	05	52

- There are 400 digits in this random number
- table. 3 appears 41 times

3 FACTORS, 2 LEVELS



		A1	A2
B1	C1	y_2, y_{12}	y_5, y_{11}
	C2	y_8, y_{10}	y_4, y_7
B2	C1	y_6, y_{16}	y_9, y_{14}
	C2	y_1, y_{13}	y_{15}, y_3

- Four dimensional visibility with a $2^3 = 8$ test combination full factorial matrix

LABEL THE CELLS

8 Test
Combination

2^3

		A-	A+
C-	B-	(1)	a
C-	B+	b	ab
C+	B-	c	ac
C+	B+	bc	abc

YATES NOTATION

$2^3 = 8$ Test Combination

Cell	A	B	AB	C	AC	BC	ABC
(1)	-	-	+	-	+	+	-
a	+	-	-	-	-	+	+
b	-	+	-	-	+	-	+
ab	+	+	+	-	-	-	-
c	-	-	+	+	-	-	+
ac	+	-	-	+	+	-	-
bc	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+

YATES' NOTATION

$$2^4 = 16 \quad \text{Test Combination}$$

Cell	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD	y
(1)	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	•
a	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	•
b	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	•
ab	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	•
c	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	•
ac	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	•
bc	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	•
abc	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	•
d	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	•
ad	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	•
bd	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	•
abd	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	•
cd	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	•
acd	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	•
bcd	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	•
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	•

YATES' WORK SESSION

Y = Yield Strength , PSI

A , B and C are Concentrations of 3 Separate Elements

		A-	A+
C-	B-	58 56	36 39
	B+	51 53	34 32
C+	B-	53 48	54 59
	B+	49 49	55 61

Determine the size of each contrast using Yates' Algorithm

What combination of elements will give the highest yield strength?

THE ALGORITHM

Two variables ; A , B

∴ Number of Variables , n = 2 Number of columns , N = n = 2

For Top ½ of Each Column : $1^{st} + 2^{nd}$

Cell Name	$\bar{y}$	Column Number 1	Column Number 2	$\Delta\bar{y}$'s	Contrast
(1)	2	7	20		
a	5	13	8	4	A
b	4	3	6	3	B
ab	9	5	2	1	AB
Total	20				

For Bottom 1/2 Of Each Column: $2^{nd} - 1^{st}$

(1)	2	7	20		
a	5	13	8	4	A
b	4	3	6	3	B
ab	9	5	2	1	AB

YATES' WORK SESSION

Cell	y	y	ȳ	1	2	3	÷ 4	Rank
(1)	58	56	57	94.5	179.5	393.5		
a	36	39	37.5	85	214	-23.5	-5.9	3
b	51	53	52	107	-38.5	-9.5	-2.4	
ab	34	32	33	107	15	3.5	.9	
c	53	48	50.5	-19.5	-9.5	34.5	8.6	2
ac	54	59	56.5	-19	0	53.5	13.4	1
bc	49	49	49	6	0.5	9.5	2.4	
abc	55	61	58	9	3	2.5	.6	
		Total	393.5					

YATES' WORKSHEET , 3 VARIABLES

Cell	y	y	$\bar{Y}$	1	2	3	÷4	RANK
(1)								
a								
b								
ab								
c								
ac								
bc								
abc								
		TOTAL						

一般實驗設計進行之流程/步驟
(Program Flowchart for Experiment Design)

Basically , A twelve-steps approach for conducting any experiment design proposed by Schmidt can be divided into the following three stages (Pan):

STAGE1

(準備及設計選擇階段):
Defined the problems and state the objective of the Experiment ; Select quality characteristics(response) and input variables (factors) ; Determine the desired number of runs and replications ; consider the randomization of runs during the selection of the best design type.

STAGE2 (實驗及分析資料階段)：

Conduct the experiment and record the data ; Analyze the data using Analyze of mean , Analysis of variance/ Yate’s Algorithm and Normal Probability Plot to determine the significant main and interaction effects

↓

STAGE3 (建立預估模式及確認評估階段):

Develop a fitted model using regression analysis ; Draw conclusion and make prediction . Perform confirmatory tests , Assess results and make decision.

ANALYSIS OF VARIATION AND ESTIMATION FOR a A X B FACTORIAL EXPERIMENT

因子實驗設計之ANOVA分析:

THE TOTAL SUM OF SQUARES CAN BE PARTITIONED INTO :

TOTAL SS = SS(A) + SS(B) + SS(AB) +SSE

ANOVA TABLE FOR A × B FACTORIAL EXPERIMENT			
SOURCE	d.f.	SS	MS
FACTOR A	(a-1)	SS(A)	SS(A)/(a-1)
FACTOR B	(b-1)	SS(B)	SS(B)/(b-1)
INTERACTION AB	(a-1)(b-1)	SS(AB)	SS(AB)/((a-1)(b-1))
ERROR	(n-ab)	SSE	SSE/(n-ab)
TOTAL	(n-1)	TOTAL SS	

THE COMPUTATION FORMULAS FOR THE APPROPRIATE SUM OF SQUARES ARE: TOTAL SS = $\sum(EACH\ \ OBSERVATION)^2 - CF$

WHERE $CF = \dfrac{(TOTAL\ OF\ \ ALL\ OBSERVATION)^2}{rab}$

AND N = rab

r = NUMBER OF TIMES EACH FACTORIAL TRESTMENT COMBINATION APPEARS IN THE EXPERIMENT

A X B FACTORIAL EXPERIMEN (CONTINUED)

$$SS(A) = \frac{\sum A^2}{rb} - CF$$

$$SS(B) = \frac{\sum B^2}{ra} - CF$$

$$SS(AB) = \frac{\sum (AB)^2}{r} - CF - SS(A) - SS(B)$$

$$SSE = TOTAL\ \ SS - SSA - SSB - SS(AB)$$

TEST EACH NULL HUPOTHESIS:

$$F = \frac{MS(A)}{MSE}\ \ AND\ \ F = \frac{MS(B)}{MSE}\ \ AND\ \ F = \frac{MS(AB)}{MSE}$$

EXAMPLE: A X B FACTORIAL EXPERIMENT

THE EVALUATION OF A FLAME RETARDANT WAS CONDUCTED AT TWO DIFFERENT LABORATORIES ON THREE DIFFERENT MATERIALS WITH THE FOLLOWING RESULTS

	MATERIALS		
LABORATORY	1	2	3
1	4.1 , 3.9 4.3	3.1 , 2.8 3.3	3.5 , 3.2 3.6
2	2.7 , 3.1 2.6	1.9 , 2.2 2.3	2.7 , 2.3 2.5

EXAMPLES: A X B FACTORIAL EXPERIMENT (CONTINUED)

TOTAL FOR CALCULATING SUMS OF SQUARES				
	MATERUAL(B)			
LABORATORY	1	2	3	TOTAL(A)
1 2	12.3 8.4	9.2 6.4	10.3 7.5	31.8 22.3
TOTAL(B)	20.7	15.6	17.8	54.1

THERE ARE N = rab = (3)(2)(3) =18 OBSERVATION

$$CF = \frac{(54.1)^2}{18} = 162.6006$$

$$\text{TOTAL SS} = (4.1^2 + 3.9^2 + \ldots + 2.5^2) - CF$$
$$= 170.53 - 162.6006 = 7.9294$$

$$\text{SS(A)} = \frac{(31.8^2 + 22.3^2)}{9} - CF$$
$$= 167.6144 - 162.6006 = 5.0139$$

$$\text{SS(B)} = \frac{(20.7^2 + 15.6^2 + 17.8^2)}{6} - CF$$
$$= 164.7817 - 162.6006 = 2.1811$$

$$\text{SS(AB)} = \frac{(12.3^2 + 9.2^2 + \ldots + 7.5^2)}{3} - CF - SS(A) - SS(B)$$
$$= 169.93 - 162.6006 - 5.0139 - 2.1311 = .1344$$

$$\text{SSE} = TOTAL\ SS - SS(A) - SS(B) - SS(AB)$$
$$= 7.9294 - 5.0139 - 2.1811 - .1344 = .6000$$

EXAMPLES: A X B FACTORIAL EXPERIMENT (CONTINUED)

THE FOLLOWING ANOVA TABLE APPLIES

SOURCE	d.f.	SS	MS	F
LABORATORY(A)	1	5.0139	5.0139	100.28
MATERIAL(B)	2	2.1811	1.0906	21.81
INTERACTION(AB)	2	.1344	0.672	1.34
ERROR	12	.6000	0.0500	
TOTAL	17	7.9294		

TEST THE HYPOTHESIS FOR:
NO INTERACTION

$$F = \frac{MS(AB)}{MSE} = \frac{0.0672}{0.05} = 1.34$$

SINCE $F_{0.05_{2,12}} = 3.89$, THE INTERACTION IS NOT SIGNIFICANT

THE NULL HYPOTHESIS IS NOT REJECTED

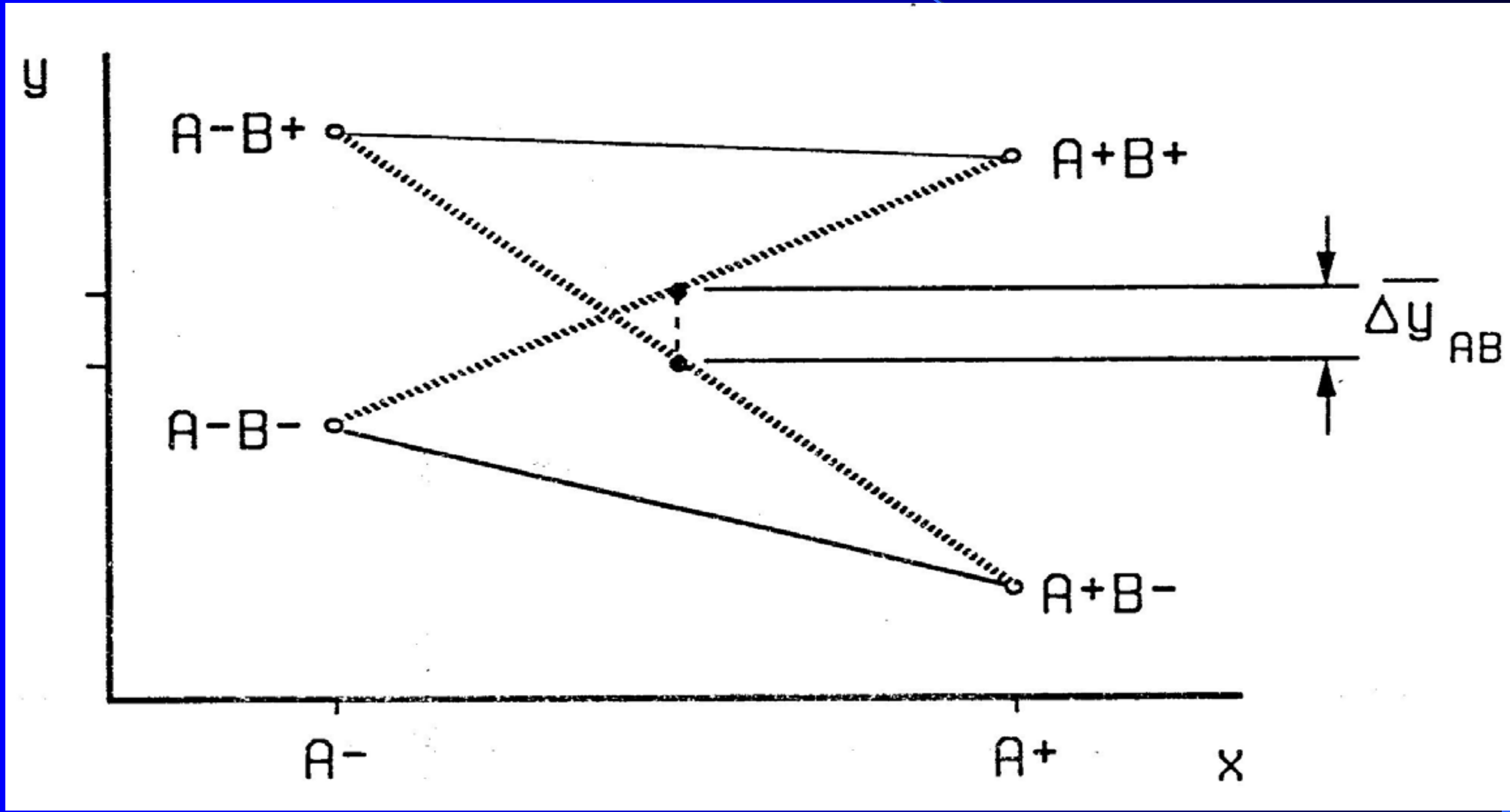
NO DIFFERENCES AMONG MATERIALS

$$F = \frac{MS(B)}{MSE} = \frac{1.0906}{0.0500} = 21.81$$

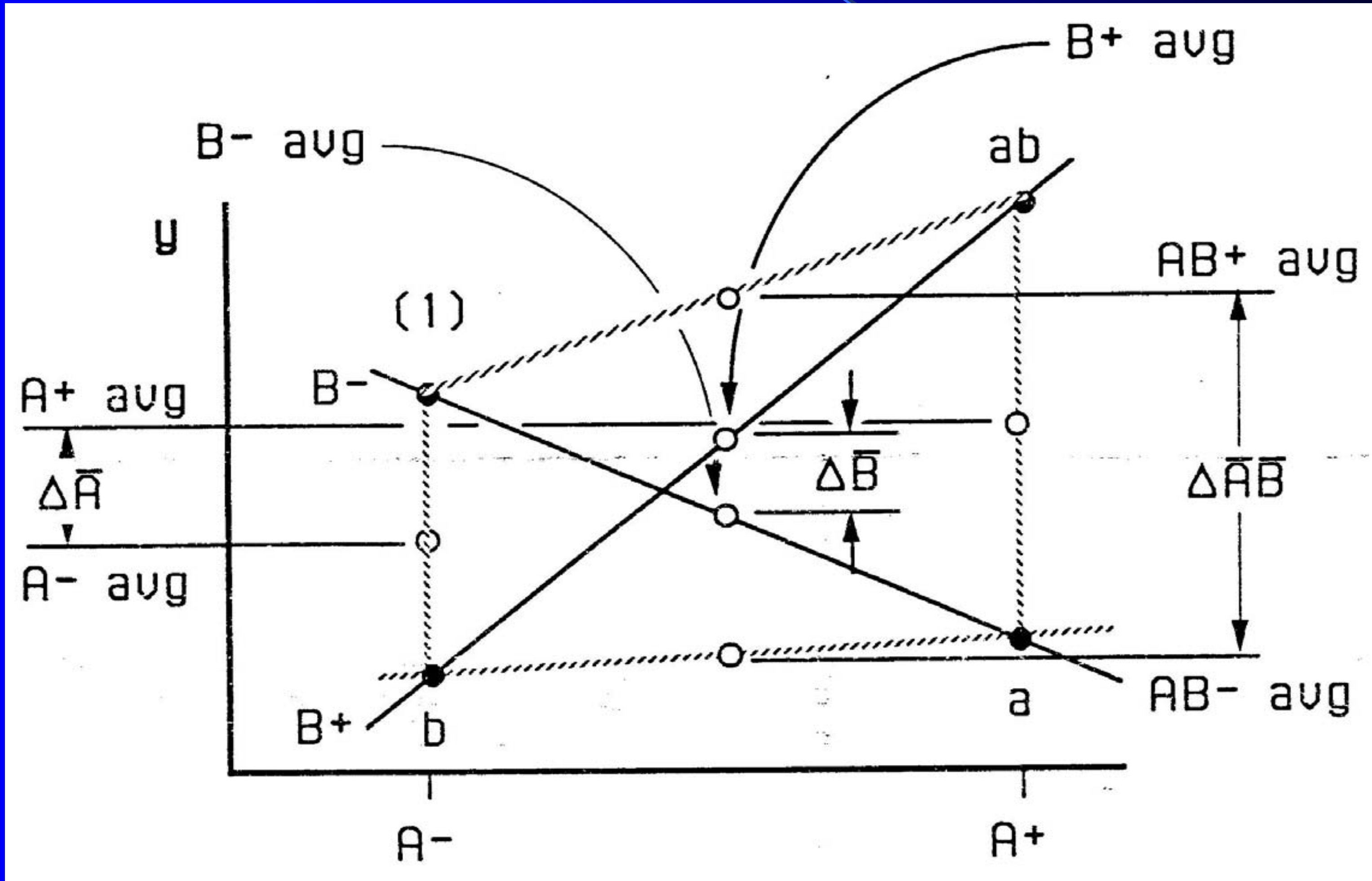
SINCE $F_{0.05_{2,12}} = 3.89$, MATERIAL IS IMPORTANT.

THE NULL HYPOTHESIS IS REJECTED

MAIN EFFECT LARGER THAT INTERACTION



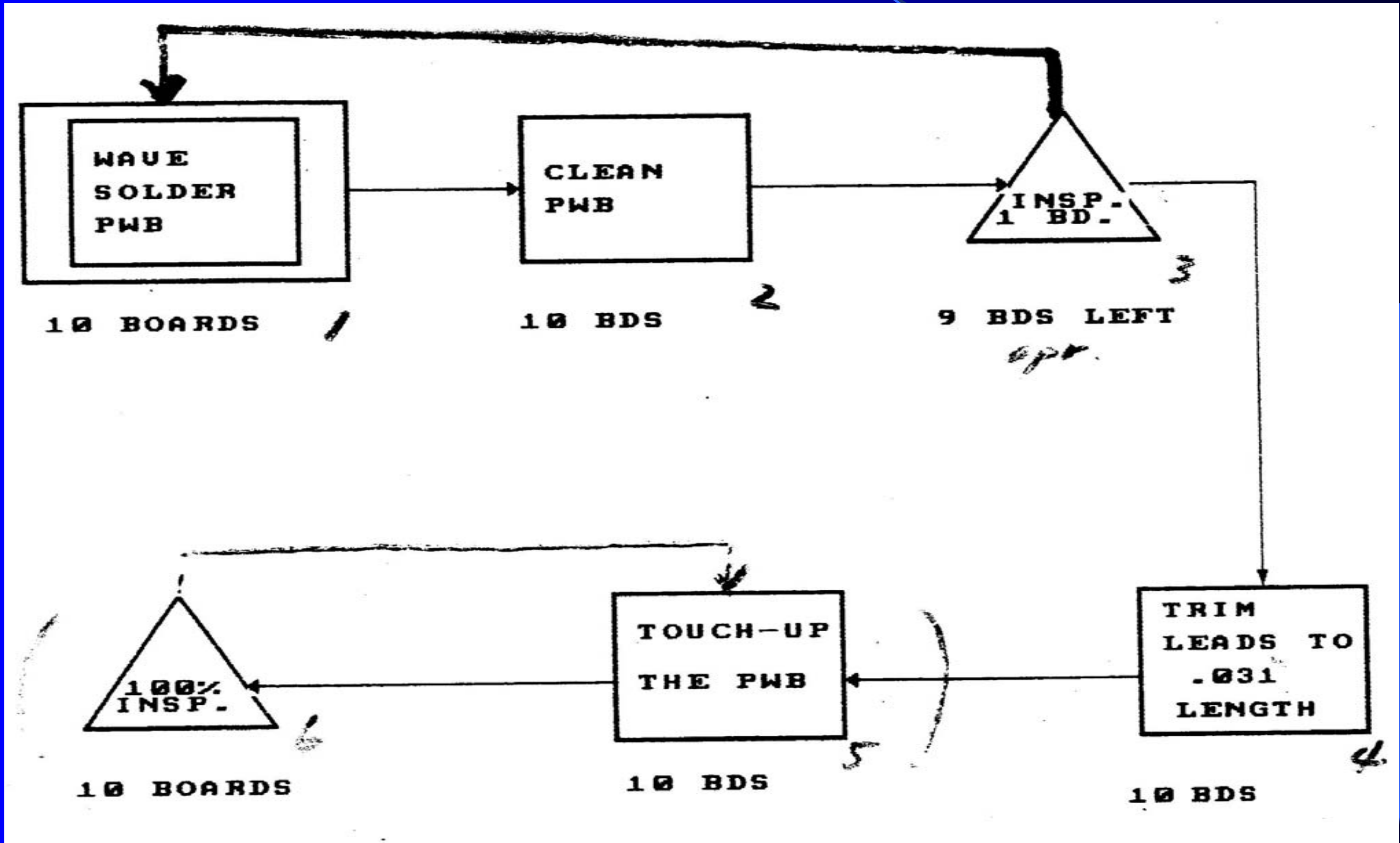
INTERACTION LARGER THAT MAIN EFFECT

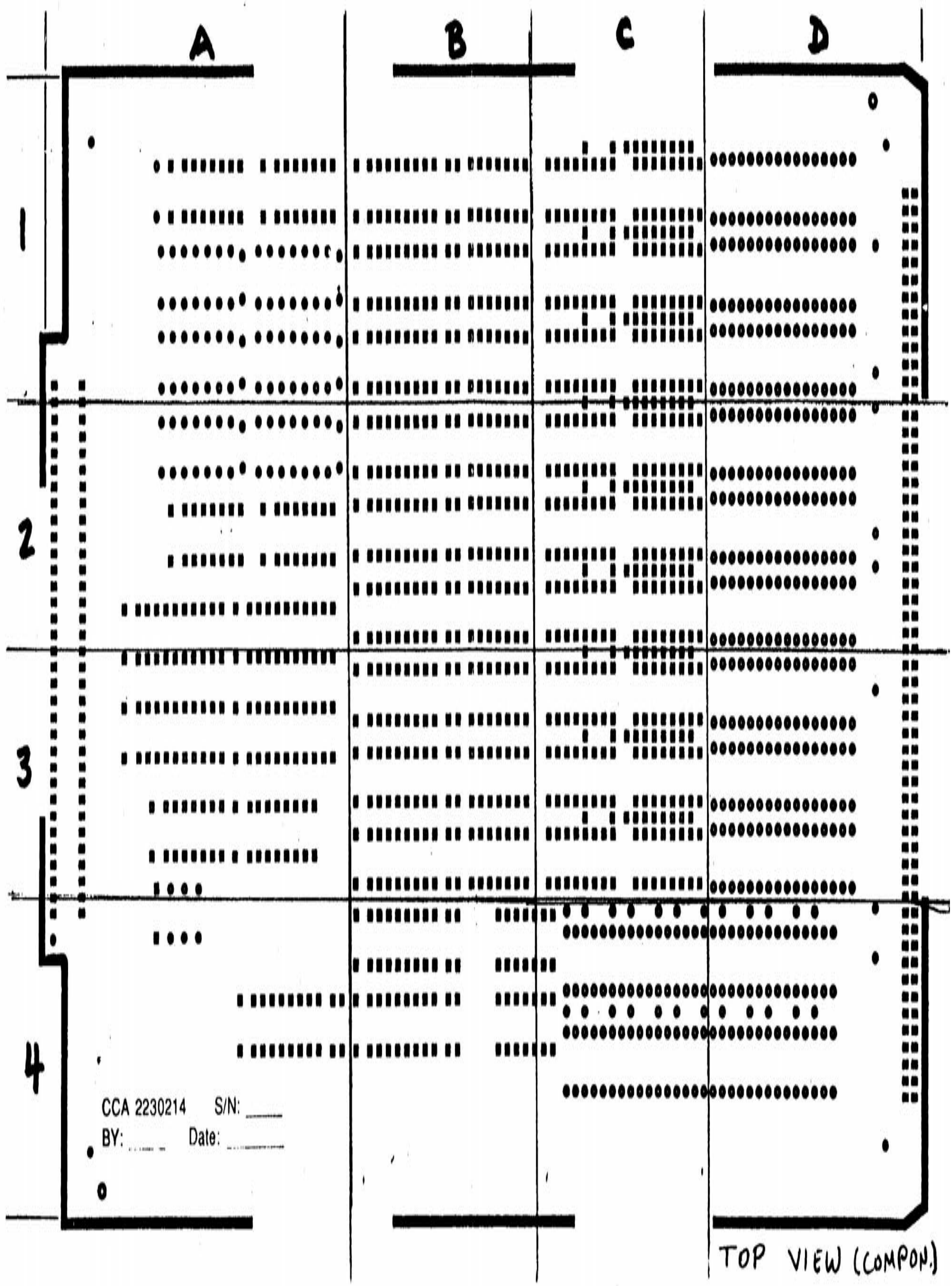


CONDUCT FULL FACTORIAL EXPERIMENT TO WAVE SOLDER PROCESS AT TELEDYNE

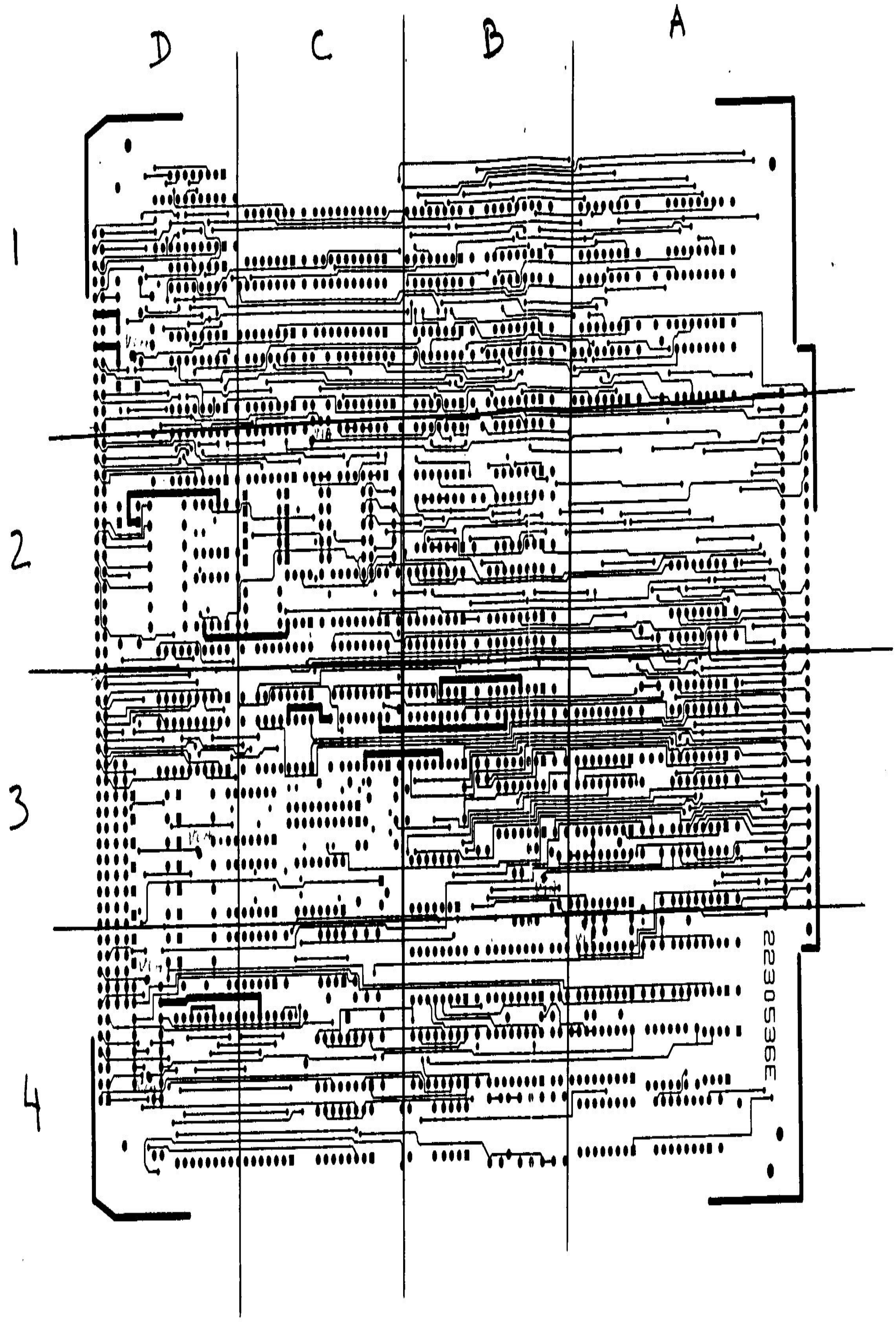
- OBJECTIVE : TO DETERMINE THE EFFECT OF FLUX TYPE AND LEAD LENGTH ON THE DFDAU WAVE SOLDERING DEFECTS
- PLANNED STEPS FOR STATISTICALLY DESIGNED EXPERIMENT
- (1). SELECT OUTPUT VARIABLES , 2 FACTORS , 2 LEVELS 8 RUNS
- (2). RANDOMIZE THE SEQUENCE OF RUNS AND LABLE 8 DFDAU BDS
- (3). SELECT TWO TOUCHUP OPR. TO INSPECT VARIOUS WS DEFECTS
- (4). ISOPLOT THE MAJOR WS DEFECTS FOR TOP/REAR SIDES TO COMPARE ONE OPERATOR AGAINST ANOTHER
- (5). ANALYZE THE DATA USING ANOVA TABLES WITH INTERACTIONS OR YATES ANALYSIS TABLE
- (6). PLOT/INTERPRET THE RESULTS AND DRAW THE CONCLUSIONS

CURRENT WAVE SOLDERING PROCESS FLOW CHART





TOP VIEW (COMPON.)



SOLDER DEFECTS
1-SOLDER VOID
2-SOLDER PEAK
3-INSUFICIENT SOLDER (SOLDER SIDE)
4-INSUFICIENT SOLDER (COMPON. SIDE)
5-NON WETTING/DE-WETTING
6-UNSOLDER
7-POOR WETTING
8-EXCESSIVE SOLDER
9-DISTURBED/COLDER SOLDER
10-VIA
11-SOLDER BRIDGE
12-OTHER

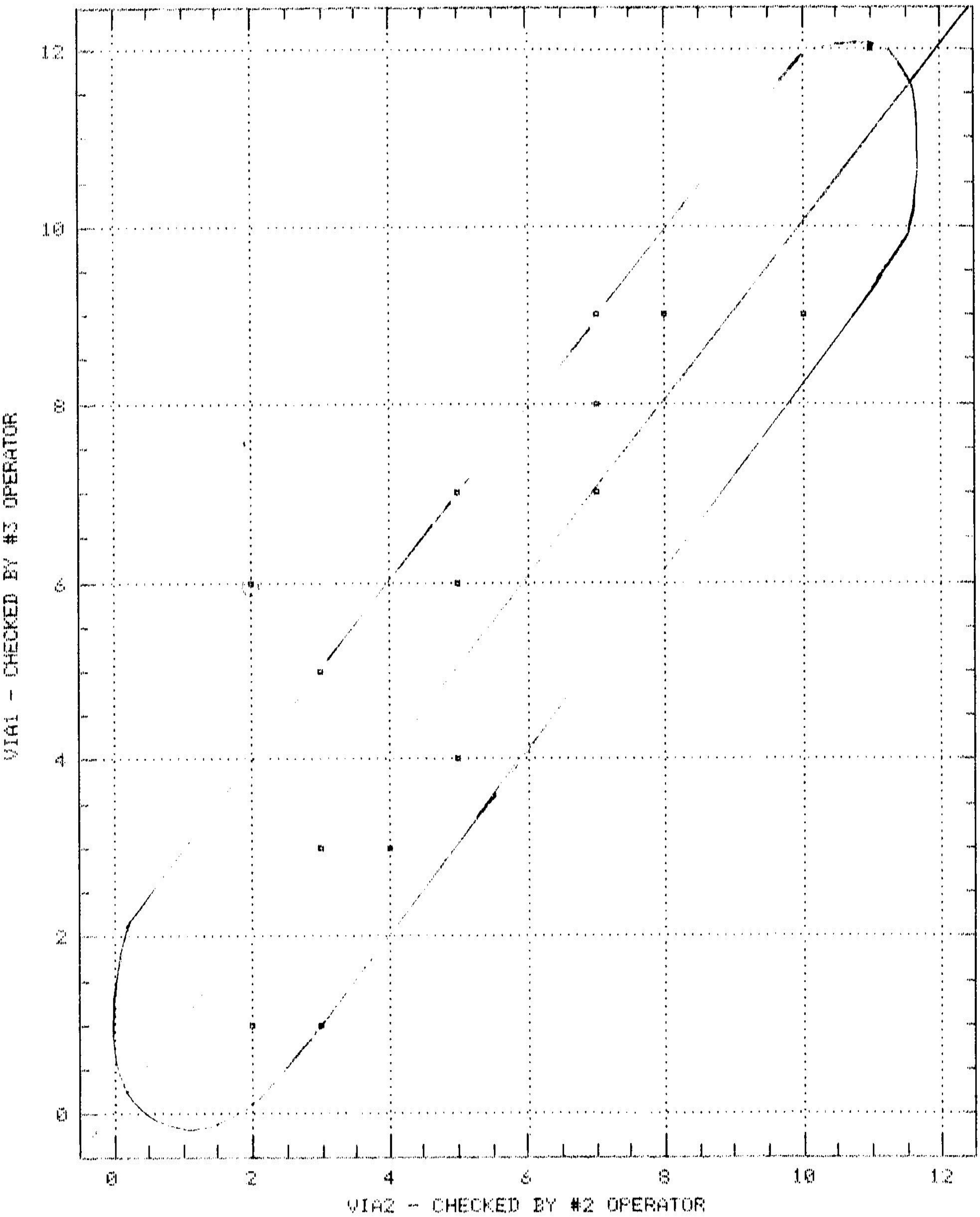
DEFECT#	QTY
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	7
11	
Other	
TOTAL	

CCA 2230535 S/N: Y4
BY: ALu Date: 3-25-91 Solder Time: ____

REAR SIDE
LAYER 8

SIZE	CODE IDENT	DRAWING NO.
B	98571	2230536E
SCALE: 1/1		SHEET 9

ISOPLOT OF WAVE SOLDERING DEFECT - VIA HOLE
COMPARISON STUDY - INSPECTED BY OPERATOR #1 AND #2



$\Delta M = 28$ mm

$\lambda = 172$ mm

$\Delta P = \sqrt{\frac{172^2}{2} - \frac{28^2}{2}}$

$= 120$

$\Delta P / \Delta M = 120/28 = 4.3$

STATISTICALLY DESIGNED EXPERIMENT

Serial No.	Flux Type	Lead Length	Label
Y1	New(OA)	Trimmed Leads	ab
Y2	Old(RMA)	Std Lead Length	(1)
Y3	Old(RMA)	Std Lead Length	(1)
Y4	New(OA)	Std Lead Length	b
Y5	Old(RMA)	Trimmed Leads	a
Y6	New(OA)	Trimmed Leads	ab
Y7	New(OA)	Std Lead Length	b
Y8	Old(RMA)	Trimmed Leads	a

New Flux = Alpha # 857

Old Flux = RMA

Std Trimmed Lead Length = as they come out of Prep. Room. IC , Conn. Not Trimmed.(接點處之引線未被切平)

Trimmed Leads = about .045”

YATES' ALGORITH

NOTATION

$$2^2 = 4 \; Test \;\; Combinations$$

W/O Leads Trimmed

	A-	A+
Old Flux B-	(1)	a
New Flux B+	b	ab

ANOVA TABLE

	Contrasts		
Cell	A	B	AB
(1)	-	-	+
a	+	-	-
b	-	+	-
ab	+	+	+

OPERATOR #1

	W/T Leads	T Leads
Old Flux	12 10.5 9	9 7.5 6
New Flux	7 8 9	8 7 6

OPERATOR #2

Old Flux	11 9.5 8	9 7 5
New Flux	7 7 7	7 4.5 2

AVG OPERATOR

	W/T Leads	T Leads
Old Flux	10.5 10 9.5	7.5 7.25 7
New Flux	8 7.5 7	7 5.75 4.5

ANOVA ANALYSIS FOR TWO FACTORIAL EXPERIMENT

Two way ANOVA for FLUXT . VIADEF

Source of Variation	Sum of Squares	D.F.	Mean Square	F-Ratio	P Value	Sign. Diff.
FLUX	8	1	8	7.52941	0.0517	2
LEADL	10.125	1	10.125	9.52941	0.0367	1
Interaction	0.5	1	0.5	0.470588	0.5373	No
Error	4.25	4	1.0625			
Total (corr.)	22.875	7				

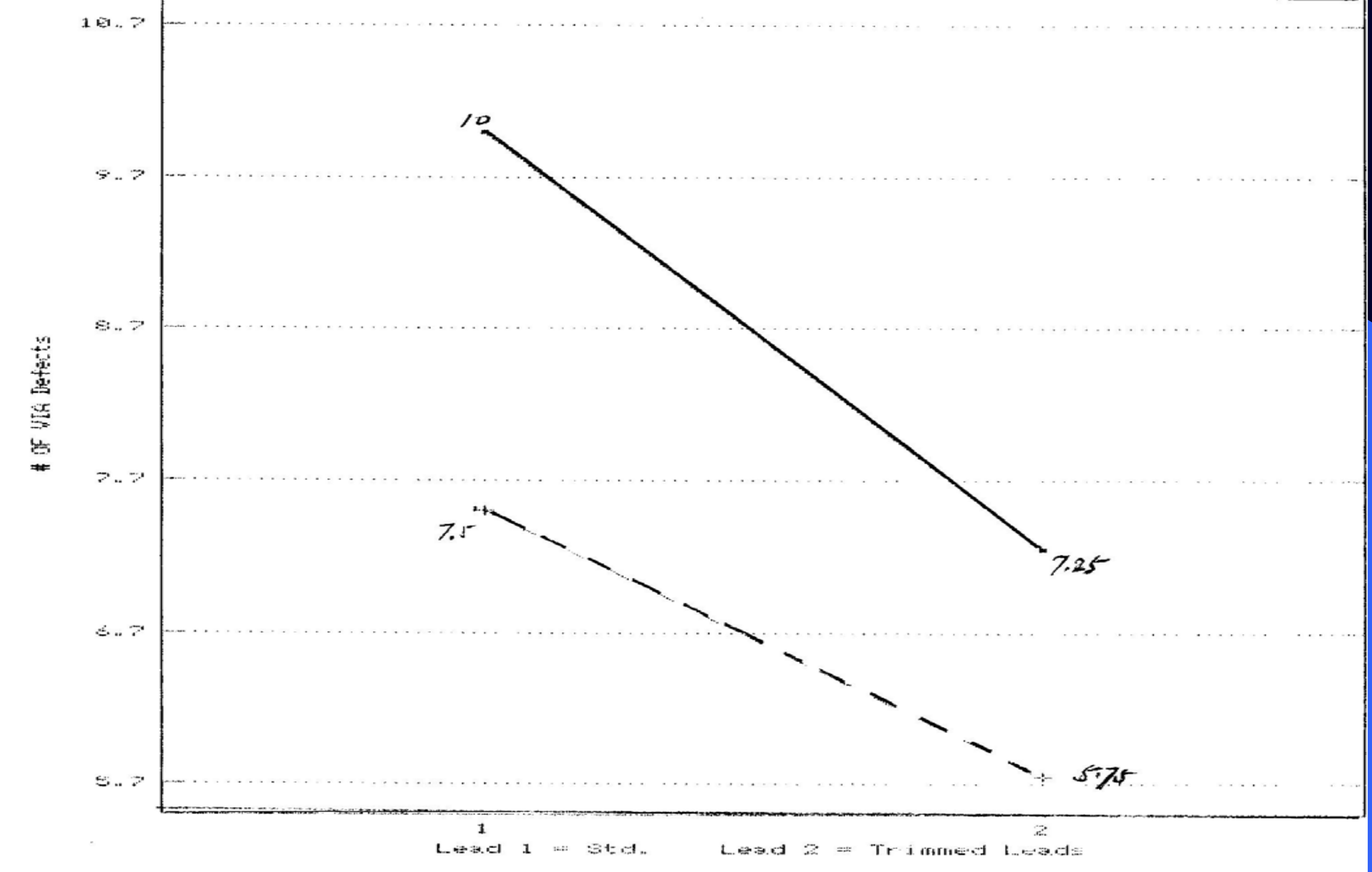
Table of Means

FLUX	Sample Size	Sample Mean	Standard Error	Estimated Effect
1	4	8.625	0.515388	1
2	4	6.625	0.515388	-1

LEADL	Sample Size	Sample Mean	Standard Error	Estimated Effect
1	4	8.75	0.515388	1.125
2	4	6.5	0.515388	-1.125

FLUX	LEADL	Sample Size	Sample Mean	Standard Error	Estimated Effect
1	1	2	10	0.728869	2.37
1	2	2	7.25	0.728869	-0.37
2	1	2	7.5	0.728869	-0.12
2	2	2	5.75	0.728869	-1.87
Overall		8	7.625	0.364434	

Interaction Plot for FLUX-LEAD LENGTH Factors
Flux 1 = OA Flux 2 = RMA



2 FACTOR FULL FACTORIAL EXPERIMENTSUMMARY AND CONCLUSION

SUMMARY OF FINDING

- ISPLOT Reveal that 2 Opr. were fairly consistent in calling out VIA Defects
- VIA Defects consist of 77 % vs 93 % of Ttl Defects , 1 vs2
- Only the rear VIA defects are considered for output measures
- Defect Level : 2941 ppm (Trimmed Leads)
- Defect Level : 3959 ppm (Std. Lead Length)

CONCLUSION

- The ANOVA/Yates Analysis show that Lead Length to be the most significant Contrast
- Interaction between Flux and Lead Length proven to be the least significant
- 26 % Improvement can expected if use the “ Trimmed Leads “
- 23 % Improvement can expected if use the “ OA “ Flux .
- Optimal region for Board Temp needs to be further studied

LATIN SQUARE (拉丁方格)

Opr. / Processes

Processes	I	II	III
1	A	B	C
2	B	C	A
3	C	A	B

Model $$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk} \quad i, j, k = 1 \ldots, r$$

Operators	I , II , III
Processes	1 , 2 , 3
Material Source	A , B , C

GRECO-LATIN SQUARE

	I	II	III
1	A_α	B_β	C_γ
2	B_γ	C_α	A_β
3	C_β	A_γ	B_α

Operators	I , II , III
Processes	1 , 2 , 3
Material # 1 Source	A , B , C
Material # 2 Source	α, β, γ

LATIN SQUARE DESIGN

By using a Latin Square design, three sources of variation, A. B. and C. can be investigated simultaneously providing there is no interaction between the three factors and also that each of them has the same number of levels r.

For example suppose each factor has four levels denoted by $A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4$ *and* C_1, C_2, C_3, C_4. If factor A is associated with the rows of the table and B with the columns of the table then each levels of factor C must appear once in each row and once in each column. In order to achieve this a systematic cyclic pattern can be set down for the C's as shown in the table. To randomize the design, the allocation of the A's and B's to the rows and columns is then carried out at random.

A_i \ B_j	B_4	B_2	B_1	B_3
A_2	C_1	C_2	C_3	C_4
A_4	C_4	C_1	C_2	C_3
A_3	C_3	C_4	C_1	C_2
A_1	C_2	C_3	C_4	C_1

Latin Square Models

Parametric
Random

$$\left.\right\} Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk} \quad i, j, k = 1\ldots, r$$

The $\alpha's$ $\beta's$ $\gamma's$ *and* $\varepsilon's$ are mutually independent.

Analysis of variance for a Latin Square design

The total sum of squares is divided into four component parts, one for each source of variation and one for the residual.

$$Y_{\ldots} = \sum\sum\sum y_{ijk}, \quad N = r^2, \quad C.F. = \frac{Y^2_{\ldots}}{N}$$

$$SST = \sum\sum\sum y^2_{ijk} - C.F.$$

$$SSA = \frac{1}{r}\sum Y^2_{i..} - C.F.$$

$$SSB = \frac{1}{r}\sum Y^2_{.j.} - C.F.$$

$$SSC = \frac{1}{r}\sum Y^2_{..k} - C.F.$$

$$SSE = SST - SSA - SSB - SSC$$

Here $Y_{i\cdot\cdot}$ is the sum of over the r observations in which factor A is at level i , with similar interpretation for $Y_{\cdot j\cdot}$ *and* $Y_{\cdot\cdot k}; Y_{\cdot\cdot\cdot}$ is the sum of all the r^2 observations.The analysis and test statistic which are the same for both models, are summarized in the following ANOVA Table.

ANOVA Table for Latin Square

Source	d.f	S.S	M.S	F
Factor A	r-1	SSA	$\hat{\sigma}_2^{\,2}$	$\hat{\sigma}_2^{\,2} / \hat{\sigma}_1^{\,2}$
Factor B	r-1	SSB	$\hat{\sigma}_3^{\,2}$	$\hat{\sigma}_3^{\,2} / \hat{\sigma}_1^{\,2}$
Factor C	r-1	SSC	$\hat{\sigma}_4^{\,2}$	$\hat{\sigma}_4^{\,2} / \hat{\sigma}_1^{\,2}$
Residual	$r^2 - 3r + 2$	SSE	$\hat{\sigma}_1^{\,2}$	
Total	$r^2 - 1$	SST		

EXAMPLE:

Analysis the following 4 X 4 Latin Square in which the effects Of three factors, farm, type of fertilizer applied, and method of application $(C_1, C_2, C_3 \ or \ C_4)$,on the yield crop are being investigated

		Fertilizer			
		B_1	B_2	B_3	B_4
Farm	A_1	$33C_4$	$33C_3$	$33C_1$	$33C_2$
	A_2	$38C_2$	$33C_1$	$37C_3$	$32C_4$
	A_3	$33C_1$	$36C_2$	$35C_4$	$32C_3$
	A_4	$32C_3$	$32C_4$	$37C_2$	$29C_1$

To ease the calculations, the data can be coded by subtracting 33 from each observation. Then the row and column totals and the totals for each method of application are calculated.(扣除33, 不致影響ANOVA分析)

		Fertilizer 1	2	3	4	Total
Farm	1	$0C_4$	$0C_3$	$0C_1$	$2C_2$	2
	2	$5C_2$	$0C_1$	$4C_3$	$-1C_4$	8
	3	$0C_1$	$3C_2$	$2C_4$	$-1C_3$	4
	4	$-1C_3$	$-1C_4$	$4C_2$	$-4C_1$	2
Total		4	2	10	-4	12
Method		C_1	C_2	C_3	C_4	
Total		-4	14	2	10	12

$$Y... = 12 \qquad N = r^2 = 16$$

$$C.F. = {12^2}/{16} = 9.0$$

$$SST = 0^2 + 0^2 + 0^2 + 2^2 + ...(-4)^2 - C.F. = 94 - 9 = 85$$

$$SSA = [2^2 + 8^2 + 4^2 + (-2)^2]/4 - C.F. = 22 - 9 = 13$$

$$SSB = [4^2 + 2^2 + 10^2 + (-4)^2]/4 - C.F. = 34 - 9 = 25$$

$$SSC = [(-4)^2 + 14^2 + 2^2 + 0^2]/4 - C.F. = 54 - 9 = 45$$

$$SSE = SST - SSA - SSB - SSC = 2$$

The calculations necessary for testing the significant of the three factors are summarized in the following ANOVA table.

Source	d.f	S.S	M.S	F
Farm	3	13	4.33	13.0
Fertilizer	3	25	8.33	25.0
Method	3	45	15.00	45.0
Residual	6	2	0.333	
Total	15	85		

Since the critical value are $F_{0.99}(3,6) = 9.78$

and $F_{0.999}(3,6) = 23.70$, the farm effect is significant at 1 % level

The type of fertilizer used and the method of application are both significant at the 0.1 % level .

DEVELOPING A TWO-LEVEL FRACTIONAL FACTORIAL

A 2^3 Full Factorial Experiment

Cell	A	B	AB	C	AC	BC	ABC
(1)	-	-	+	-	+	+	-
a	+	-	-	-	-	+	+
b	-	+	-	-	+	-	+
ab	+	+	+	-	-	-	-
c	-	-	+	+	-	-	+
ac	+	-	-	+	+	-	-
bc	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+

Starting the Plan for
A 2^{4-1} Fractional Factorial Experiment

Cell	A	B	D	C	AC	BC	ABC
(1)	-	-	+	-	+	+	-
a	+	-	-	-	-	+	+
b	-	+	-	-	+	-	+
ab	+	+	+	-	-	-	-
c	-	-	+	+	-	-	+
ac	+	-	-	+	+	-	-
bc	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+
Alias			AB				

ALGEBRA OF SIGNS

Axioms

1. Anything Squared Is A (+)

$$(+)^2 = I$$

$$(-)^2 = I$$

2. A (+) Times Anything Changes Nothing

$$(+) \times I = (+)$$

$$(-) \times I = (-)$$

Example

sign D = sign AB

$\therefore$ in sign algebra D = AB

To find the alias for A, eliminate B:

Multiply both sides by the sign of B

$$\therefore BD = AB^2$$

$$\therefore B^2 = + \quad by \quad Axiom \quad \#1$$

And + changes nothing by Axiom # 2

BD = A

An alias of A is the BD interaction

(the sign of BD will always match the sign of A)

What is the alias for B?

A 2^{4-1} Fractional Factorial Experiment

Cell	A	B	D	C	AC	BC	ABC
(1)	-	-	+	-	+	+	-
a	+	-	-	-	-	+	+
b	-	+	-	-	+	-	+
ab	+	+	+	-	-	-	-
c	-	-	+	+	-	-	+
ac	+	-	-	+	+	-	-
bc	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+
Alias	BD	AD	AB	ABCD	BCD	ACD	CD

$$I = ABD \ \therefore\ ABD \text{ is lost and gone for ever}$$

ALIASES FROM THE GENERATOR EQUATION

$I = ABD$

A	$A = A^2BD = BD$
B	$B = AB^2D = AD$
AB	$AB = A^2B^2D = D$
C	$C = ABCD$
AC	$AC = A^2BCD = BCD$
BC	$BC = AB^2CD = ACD$
ABC	$ABC = A^2B^2CD = CD$

4 VARIABLES, AT 2 LEVELS

$$Test\ Combinations = L^n = 2^4 = 16$$

$$Number\ of\ Contrasts = L^n - 1 = 15$$

2^{4-1} *(same size as a* 2^3 *)*

Contrasts	7
Aliases	7
Identities	1
Total	15

A 2^{5-2} FRACTIONAL FACTORIAL EXPERIMENT:

IDENTITIES AND ALIASES

Cell	A	B	D	C	E	BC	ABC
(1)	-	-	+	-	+	+	-
a	+	-	-	-	-	+	+
b	-	+	-	-	+	-	+
ab	+	+	+	-	-	-	-
c	-	-	+	+	-	-	+
ac	+	-	-	+	+	-	-
bc	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+
Alias			AB		AC		

$I_1 = ABD \quad I_2 = ACE \quad I_3 = ABD \times ACE = BCDE$

A	BD	CE	ABCDE
B	AD	ABCE	CDE
D	AB	ACED	BCE
C	ABCD	AE	BDE
E	ABDE	AC	BCD
BC	ACD	ABE	DE
ABC	CD	BE	ADE

A 2^{5-2} FRACTIONAL FACTORIAL EXPERIMENT:

Cell	A	B	D	C	E	BC	ABC
(1)	-	-	+	-	+	+	-
a	+	-	-	-	-	+	+
b	-	+	-	-	+	-	+
ab	+	+	+	-	-	-	-
c	-	-	+	+	-	-	+
ac	+	-	-	+	+	-	-
bc	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+
Alias	BD CE ABCDE	AD ABCE CDE	AB ACED BCE	ABCD AE BDE	ABDE AC BCD	ACD ABE DE	CD BE ADE

A, B, D, C , and E 5 *factors*

$2^5 - 1 = 32 - 1 = 31$ *Contrasts*

2^{5-2} *(same size as a* 2^3 *)*

Contrasts	7
Aliases	21
Identities (lost)	3
Total	31

DEFINING RELATION AND GENERATING FUNCTIONS

Fractional Factorial (1/8 fraction)

Generators:

E = ABC,   F = BCD, G = ABD

Defining Relation:

I = ABCE = BCDF = ABDG = ADEF

= CDEG = ACFG = BEFG

	Factors						
Run	A	B	C	D	E=ABC	F=BCD	G=ABD
1	+	+	+	+	+	+	+
2	+	+	+	−	+	−	−
3	+	+	−	+	−	−	+
4	+	+	−	−	−	+	−
5	+	−	+	+	−	−	−
6	+	−	+	−	−	+	+
7	+	−	−	+	+	+	−
8	+	−	−	−	+	−	+
9	−	+	+	+	−	+	−
10	−	+	+	−	−	−	+
11	−	+	−	+	+	−	−
12	−	+	−	−	+	+	+
13	−	−	+	+	+	−	+
14	−	−	+	−	+	+	−
15	−	−	−	+	−	+	+
16	−	−	−	−	−	−	−

DEFINING RELATION

- A relationship used to show the confounded sets of factors in a fractional design
- I = ABCD

Effect	Alias
A	BCD
B	ACD
C	ABD
D	ABC
AB	CD
AC	BD
AD	BC
BC	AD
BD	AC
CD	AB
ABC	D
ABD	C
ACD	B
BCD	A
ABCD	I

- 實驗之解析度(Resolution)
- THE LENGTH OF THE SHORTEST WORD IN THE DEFINING RELATION OF A TWO-LEVEL DESIGN.

R(V): Unconfounded main effects and 2-way Interactions (Unsaturated design)

R(IV): Unconfounded main effects, but 2-way interactions are confounded with other 2-way interactions (Unsaturated design)

R(III): Main effects are confounded with 2-way interactions (Saturated design)

R(II): Main effects are confounded with other main effects (Supersaturated design)

MINIMUM CONFOUNDING

Cell	A	B	AB	C	AC	BC	D
(1)	-	-	+	-	+	+	-
a	+	-	-	-	-	+	+
b	-	+	-	-	+	-	+
ab	+	+	+	-	-	-	-
c	-	-	+	+	-	-	+
ac	+	-	-	+	+	-	-
bc	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+
Alias	BCD	ACD	CD	ABD	BD	AD	ABC

I = ABCD ∴ ABCD is lost and gone forever

Only the 4 factor interaction is lost. Main effects and two factor Interactions can be separated by running another experiment.

2^{k-1} 實驗設計進行之流程 (2^{k-1} Fractional Factorial Experiment Design)

Basically, the strategic arrangement for 2^{k-1} Fractional Factorial Experiment Design can be divided into the following three stages:

STAGE1 (準備及設計選擇階段)

Select k input variables/factors and two (High and Low) levels which are anticipated to have an effect on the responses. Also select responses/output variables.

STAGE2

(實驗及分析資料階段)

Conduct 2^{k-1} experiment run, collect and record the data, then perform Yates's Algorithm and/or Analysis of Variance to estimate the effects of input variables and determine the significant effects using Normal Probability Plot.

STAGE3

(建立預估模式及確認評估階段)

Develop a fitted model for the responses. Draw conclusions and make predictions; perform confirmatory tests. Assess results and make decisions/recommendations.

2^{K-1} Fraction Factorial 案例研究 （花生油）

I.

Variable	Name	Level Low(-)	Level High(+)
A	CO_2 Pressure	415	550
B	Temperature (°C)	25	95
C	Moisture (%)	5	15
D	Flow Rate (L/min.)	40	60
E	Avg. Size (mm)	4.05	1.28

ABCD → E

Or I=ABCDE（Using CO_2 to extract oil from peanuts）

Response Variable

Y_1 : Oil Solubility (S) : Amount of oil dissolute in CO_2 (mg oil / liter CO_2)

Y_2 : Total yield of oil per batch (Y)

II.

Conduct 16 run experiment, collect Solubility and Yield data. Then, perform Yate's Algorithm and ANOVA as shown in Table 2, 3, 4.

III.

Build a fitted model for prediction（S and Y）

$$\hat{Y}_1 = b_0 + b_1 \cdot X_1 + b_2 \cdot X_2 + b_{12} \cdot X_1 X_2 \text{, where}$$

$$\hat{b}_0 = \frac{Y_1 + Y_2 + \cdots}{16}$$

$$\hat{b}_1 = \frac{l_1}{2}$$

$$\hat{b}_2 = \frac{l_2}{2}$$

$$\hat{b}_{12} = \frac{l_{12}}{2}$$

$$S = 55 + \frac{49.3}{2} \cdot X_1 + \frac{51.8}{2} \cdot X_2 + \frac{40.1}{2} \cdot X_1 X_2$$

Use coded transformation $X_1 = \dfrac{r - (r_{-1} + r_{+1})/2}{(r_{+1} - r_{-1})/2}$

$$S = 55 + \frac{49.3}{2} \cdot \left(\frac{P - 482}{67.5}\right) + \frac{51.8}{2} \cdot \left(\frac{T - 60}{35}\right) + \frac{40.1}{2} \cdot \left(\frac{P - 482}{67.5}\right)\left(\frac{T - 60}{35}\right)$$

同理， $Y = 54 - \dfrac{44}{2} \times \left(\dfrac{S - 2.67}{1.38}\right) + \dfrac{20}{2} \times \left(\dfrac{T - 60}{35}\right)$

Table 1 Half-Factorial Design

Exp. no.	A Pressure (bar)	B Temperature (°C)	C Moisture (% by wt)	D Flow rate (liters/min)	E Avg. size (mm)
1	415	25	5	40	1.28
2	550	25	5	40	4.05
3	415	95	5	40	4.05
4	550	95	5	40	1.28
5	415	25	15	40	4.05
6	550	25	15	40	1.28
7	415	95	15	40	1.28
8	550	95	15	40	4.05
9	415	25	5	60	4.05
10	550	25	5	60	1.28
11	415	95	5	60	1.28
12	550	95	5	60	4.05
13	415	25	15	60	1.28
14	550	25	15	60	4.05
15	415	95	15	60	4.05
16	550	95	15	60	1.28

Table 2 Analysis of Solubility Data

Exp. no.	Effect measured	Obs.	1	2	3	4	Mean square effect	Sum of squares
1	—	29.2	52.2	228.9	474.5	879.3	—	—
2	A	23.0	176.7	245.6	404.8	394.7	49.3	9737
3	B	37.0	61.6	139.5	210.3	414.3	51.8	10728
4	AB	139.7	184.0	265.3	184.4	320.9	40.1	6436
5	C	23.3	59.6	96.5	246.9	142.5	17.8	1269
6	AC	38.3	79.9	113.8	167.4	137.5	17.2	1182
7	BC	42.6	59.1	32.1	192.7	124.7	15.6	975
8	-DE	141.4	206.2	152.3	128.2	98.1	12.3	601
9	D	22.4	−6.2	124.5	16.7	−69.7	−8.7	304
10	AD	37.2	102.7	122.4	125.8	−25.9	−3.2	42
11	BD	31.3	15.0	20.3	17.3	−79.5	−9.9	395
12	-CE	48.6	98.8	147.1	120.2	−64.5	−8.1	260
13	CD	22.9	14.8	108.9	−2.1	109.1	13.6	744
14	-BE	36.2	17.3	83.8	126.8	102.9	12.9	662
15	-AE	33.6	13.3	2.5	−25.1	128.9	16.1	1038
16	-E	172.6	139.0	125.7	123.2	148.3	18.5	1375

effects. In Tables 3 and 4, this sign change is reflected in the mean square effect column.

The mean square effects of the four interactions involving factor D were averaged to obtain an estimate of the normal variation with four degrees of freedom. Then, for each of the remaining effects, an F-test was performed as follows. The mean square effect being tested was divided by the normal variation estimate. This value was compared to the value found in the tables of the F-distribution for one degree of freedom in the numerator and four degrees of freedom in the denominator (6). If the number is higher than the F-value, then the effect is significant at that level. Table 3 summarizes the results.

As expected, only a few factors had a large effect on the process. Two main effects, A and B, were significant ($P < 0.01$), and the A-B two-factor interaction was also significant ($P < 0.025$). No other effects were significant even at $P < 0.10$.

Because only two main effects and one two-way interaction were significant, the assumption that third- and higher order interactions are insignificant seems to be valid. Also, since the main effect D was not at all significant, the assumption that interactions involving D are insignificant also appears to be valid, and the use of these

Table 3 Solubility Data Results

Exp. no.	Effect measured	Mean square effect	Sum of squares	Degrees of freedom	F-test ratio	Significance
1	—	$\bar{x} = 55.0$	—	—	—	—
2	A	49.3	9737	1	21.9	0.01
3	B	51.8	10728	1	24.1	0.01
4	AB	40.1	6436	1	14.4	0.025
5	C	17.8	1269	1	2.8	—
6	AC	17.2	1182	1	2.7	—
7	BC	15.6	975	1	2.2	—
8	DE	−12.3	601			
9	D	−8.7	304	1	0.7	—
10	AD	−3.2	42	4		
11	BD	−9.9	395			
12	CE	8.1	260	1	0.6	—
13	CD	13.6	744			
14	BE	−12.9	662	1	1.5	—
15	AE	−16.1	1038	1	2.3	—
16	E	−18.5	1375	1	3.1	—

Table 4 Yield Results

Exp. no.	Obs.	Effect measured	Mean square effect	Sum of squares	DF	F-test ratio	Significance
1	63	—	$\bar{x} = 54.25$	—	—	—	—
2	21	A	7.25	225	1	6.2	0.10
3	36	B	19.75	1560	1	43.0	0.01
4	99	AB	5.25	110	1	3.0	—
5	24	C	1.25	6	1	0.2	—
6	66	AC	1.25	6	1	0.2	—
7	71	BC	3.0	36	1	1.0	—
8	54	DE	−3.5	49			
9	23	D	0.0	0	1	0	—
10	74	AD	−4.0	64	4		
11	80	BD	−1.75	12			
12	33	CE	6.25	156	1	4.3	—
13	63	CD	2.25	20			
14	21	BE	−0.25	0.25	1	0	—
15	44	AE	−7.0	196	1	5.4	0.10
16	96	E	−44.5	7921	1	218.5	0.01

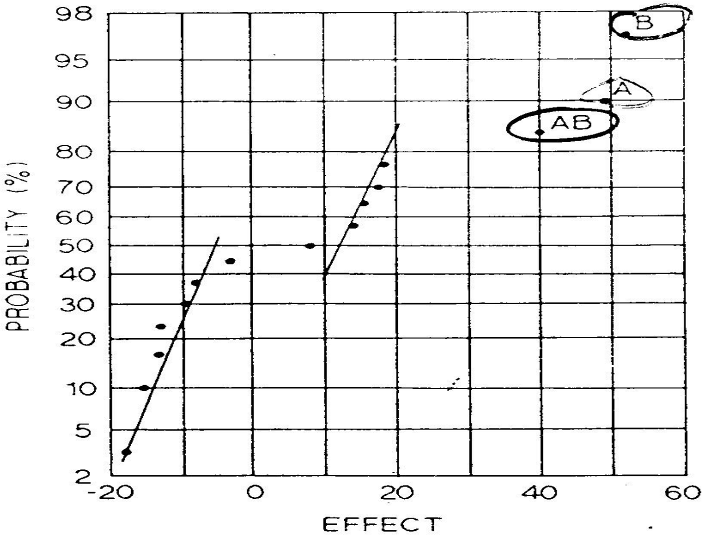


Figure 1 Normal plot of effects from solubility results of half-factorial.

effects to approximate the normal variation is appropriate.

For the yield, the analysis was performed in the same way, and the results are in Table 4. Two main effects, B and E, were significant ($P < 0.01$). A and the A-E interaction were significant at $P < 0.10$, which was not considered to be high enough for consideration here, although future experiments could explore this issue further.

The graphical test described by Box et al. (7) was used to check the numerical analysis. For this test, the effects are ordered from the smallest to largest and plotted on normal probability paper. The percentile location of each of the points is determined from the equation

$$P_i = 100 \times \frac{(i - \frac{1}{2})}{m} \quad (1)$$

for $i = 1$ to m, where m is the number of effects to be plotted. Effects due to random variation will fall roughly on a straight line, while effects that are significant will deviate from the line substantially.

Figures 1 and 2 plot the effects for the solubility and yield, respectively. They support the conclusions obtained from the numerical analysis, including the decision that the A and A-E effects on yield were not significant enough to be considered.

An interesting point to note about Figure 1 is that the effects due to random variation actually form two parallel lines which break close to the abscissa. Box and Draper (5) explain that if one experimental value is in error it could cause such a split. Any number of errors in measuring and recording the data could have been made. Because the values vary from zero, an even split will not always be found.

The magnitudes of the significant effects were used to obtain equations for the response surfaces. The procedure is illustrated using the solubility data. For each of the significant factors, the magnitude of the effect is the change in the solubility from the lower level of the factor to the high level. Therefore, half of the magnitude should be subtracted from the average solubility when the low level is used and added to the average when the high level is used. If this idea is extended to all of the significant effects, the following equation is obtained:

$$S = 55.0 + \frac{49.3}{2} \times X1 + \frac{51.8}{2} \times X2 + \frac{40.1}{2} \times X1 \times X2 \quad (2)$$

where $X1 = -1$ when the pressure is 415 bar and $+1$ when the pressure is 550 bar, and $X2 = -1$ when the temperature is 25°C and $+1$ when the temperature is 95°C. X1 and X2 can be scaled to the units of pressure and temperature by subtracting the variable from the midpoint of the levels used and then dividing that quantity by

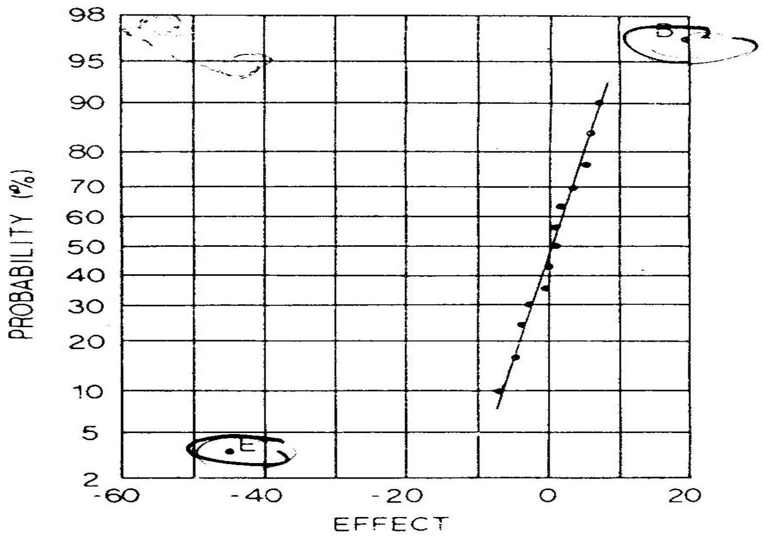


Figure 2 Normal plot of effects from yield results of half-factorial.

ANOVA for Fractional Factorial Design

Test	1	2	3	4	5	Obs
e	-	-	-	-	+	9.2
a	+	-	-	-	-	11.3
b	-	+	-	-	-	15.6
abe	+	+	-	-	+	14.2
c	-	-	+	-	-	⋮
ace	+	-	+	-	+	⋮
bce	-	+	+	-	+	⋮
abe	+	+	+	-	-	⋮
d	-	-	-	+	-	⋮
ade	+	-	-	+	+	⋮
bde	-	+	-	+	+	⋮
abd	+	+	-	+	-	⋮
cde	-	-	+	+	+	⋮
acd	+	-	+	+	-	⋮
bcd	-	+	+	+	-	⋮
abcde	+	+	+	+	+	

a	11.3	bcd	14.1
b	15.6	abe	14.2
c	12.7	ace	11.7
d	10.4	ade	9.4
e	9.2	bce	16.2
abc	11	bde	13.9
abd	8.9	cde	14.7
acd	9.6	abcde	13.2

$$\begin{cases} SS_A = \dfrac{(a - b - c + \cdots + abcde)}{2^{k-p} \cdot n} \\ n : \text{\# of replication} \end{cases}$$

$$l_A = \frac{-17.5}{8}$$

$$SST = 11.3^2 + \cdots + 13.2^2 - \frac{(196.1)^2}{16}$$

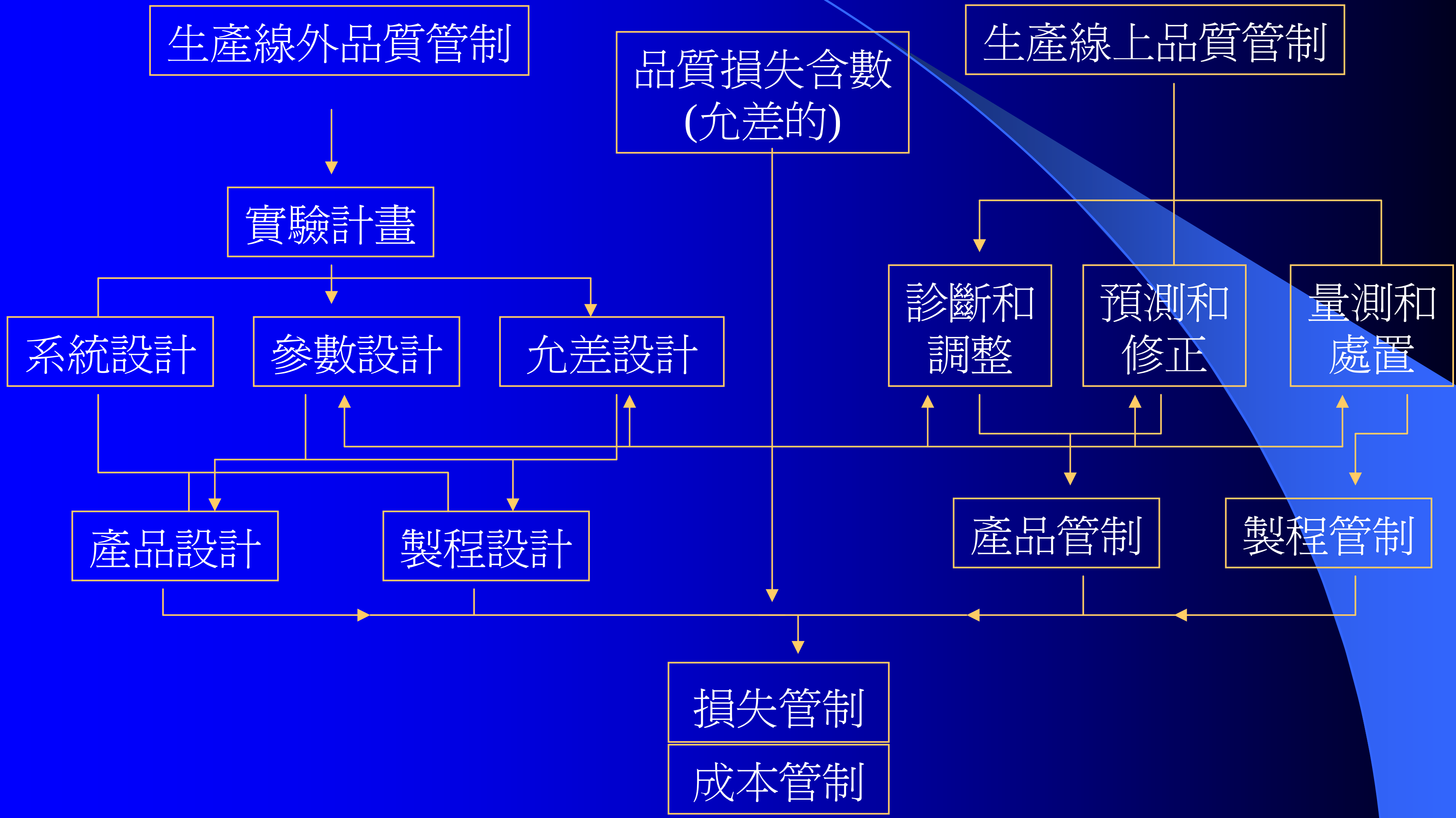
$$SS_A = \frac{(11.3 - 15.6 - 12.7 + \cdots + 13.2)^2}{2^{5-1} \cdot 1} = \frac{(-17.5)^2}{2^4} = 19.14$$

ANOVA Table

Source	SS	DOF	MS	F
A	19.14	1	19.14	6.21 *
B	20.48	1	20.48	6.65 *
C	6.63	1	6.63	2.15
D	3.71	1	3.71	1.2
E	4.95	1	4.95	1.61
Error	30.83	10	3.08	
Total	85.74	15		

田口式品質工程

目的—使用較低成本(最少之實驗次數)達到減少變異之功能

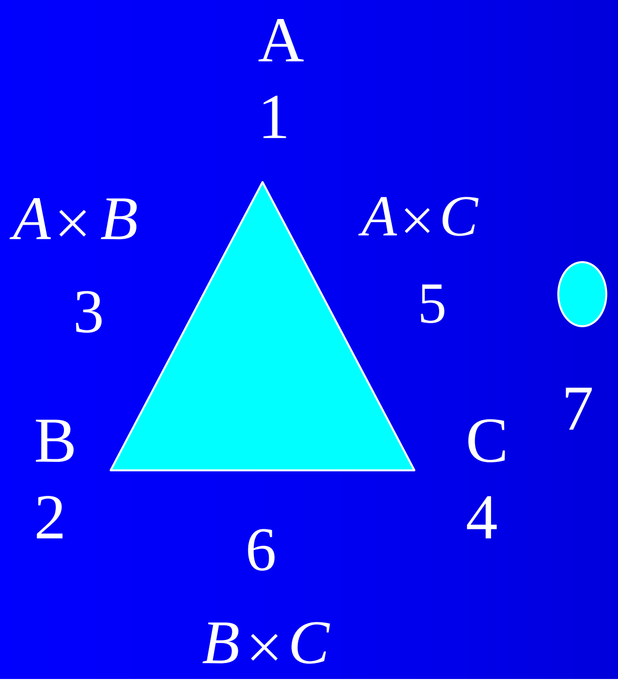


TAGUCHI DESIGNS

$L_4(2^3)$

No.	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1
	a	b	-ab

(1) Linear Graph of L_4 Table



$L_8(2^7)$

No.	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2
	a	b	-ab	c	-ac	-bc	abc

Common Orthogonal Arrays

Array	Number of Factors	Number of Levels
$L_4(2^3)$	3	2
$L_8(2^7)$	7	2
* $L_{12}(2^{11})$	11	2
$L_{16}(2^{15})$	15	2
$L_{32}(2^{31})$	31	2
$L_9(3^4)$	4	3
* $L_{18}(2^1,3^7)$	1	2
	and 7	3
$L_{27}(3^{13})$	13	3
$L_{16}(4^5)$	5	4
$L_{32}(2^1,4^9)$	1	2
$L_{36}(2^3,3^{13})$	and 9	4
$L_{64}(4^{21})$	21	4

The L_{12} and L_{18} orthogonal arrays are special designs in which interactions are generally spread across all columns. They should not be used for experiments which include the study of interactions

ORTHOGONAL ARRAY L_8

TAGUCHI NOTATION

No. \ Col.	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

YATES NOTATION

Cell Name	CONTRAST						
	C	B	BC	A	AC	AB	ABC
abc	+	+	+	+	+	+	+
bc	+	+	+	−	−	−	−
ac	+	−	−	+	+	−	−
c	+	−	−	−	−	+	+
ab	−	+	−	+	−	+	−
b	−	+	−	−	+	−	+
a	−	−	+	+	−	−	+
(1)	−	−	+	−	+	+	−

ORTHOGONAL ARRAY L_{16}

YATES NOTATION

Cell Name	CONTRAST														
	D	C	CD	B	BD	BC	BCD	A	AD	AC	ACD	AB	ABD	ABC	ABC
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
bcd	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
acd	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
cd	+	+	+	−	−	−	−	−	−	−	−	+	+	+	+
abd	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
bd	+	−	−	+	+	−	−	−	−	+	+	−	−	+	+
ad	+	−	−	−	−	+	+	+	+	−	−	−	−	+	+
d	+	−	−	−	−	+	+	−	−	+	+	+	+	−	−
abc	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
bc	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
ac	−	+	−	−	+	−	+	+	−	+	−	−	+	−	+
c	−	+	−	−	+	−	+	−	+	−	+	+	−	+	−
ab	−	−	+	+	−	−	+	+	−	−	[illegible]	+	−	−	+
b	−	−	+	+	−	−	+	−	+	+	−	−	+	+	−
a	−	−	+	−	+	+	−	+	−	−	+	−	+	+	−
(1)	−	−	+	−	+	+	−	−	+	+	−	+	−	−	+

ORTHOGONAL ARRAY $L_{16}(2^{15})$

TAGUCHI NOTATION

No. \ Col. (Factor)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

2^{7-4} Fractional Factorial Design

Test	A	B	C	D	E	F	G
(1)	-	-	-	-	-	-	-
a	+	-	-	+	+	-	+
b	-	+	-	+	-	+	+
ab	+	+	-	-	+	+	-
c	-	-	+	-	+	+	+
ac	+	-	+	+	-	+	-
bc	-	+	+	+	+	-	-
abc	+	+	+	-	-	-	+
相當田口之第	(4)	(2)	(1)	(6)	(5)	(3)	(7) 行

$L_8(2^3)$

$\therefore\ I_1$ = -124 = -135 = -236 = +1237

2^{7-4} Fractional Factorial Design (鏡射實驗)

Test	A	B	C	D	E	F	G
(1)	+	+	+	+	+	+	+
a	-	+	+	-	-	+	-
b	+	-	+	-	+	-	-
ab	-	-	+	+	-	-	+
c	+	+	-	+	-	-	-
ac	-	+	-	-	+	-	+
bc	+	-	-	-	-	+	+
abc	-	-	-	+	+	+	-
相當田口之第	(4)	(2)	(1)	(6)	(5)	(3)	(7) 行

$L_8(2^3)$

$\therefore \quad I_2 = 124 = 135 = 236 = +1237$

2^{8-4} Fractional Factorial Design

Test	1	2	3	4	5	6	7	8
(1)	-	-	-	-	-	-	-	-
a	+	-	-	-	-	+	+	+
b	-	+	-	-	+	-	+	+
ab	+	+	-	-	+	+	-	-
c	-	-	+	-	+	-	+	+
ac	+	-	+	-	+	+	-	-
bc	-	+	+	-	-	-	-	-
abc	+	+	+	-	-	+	+	+
d	-	-	-	+	+	+	-	+
ad	+	-	-	+	+	-	+	-
abd	+	+	-	+	-	-	-	+
cd	-	-	+	+	-	+	+	-
acd	+	-	+	+	-	-	-	+
bcd	-	+	+	+	+	+	+	+
abcd	+	+	+	+	+	-	+	-
相當田口之第	(8)	(4)	(2)	(1)	(7)	(9)	(14)	(15) 行

$L_{16}(2^{15})$ $\therefore$ I = 2345 = -146 = 1237 = -12348

$L_4(2^3)$

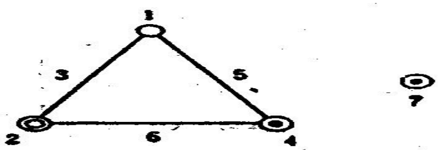
No.	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1
	a	b	ab
Group	1	2	

(1)

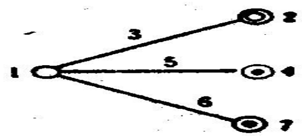
$L_8(2^7)$

No.	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2
	a	b	ab	c	ac	bc	abc
Group	1	2		3			

(1)



(2)

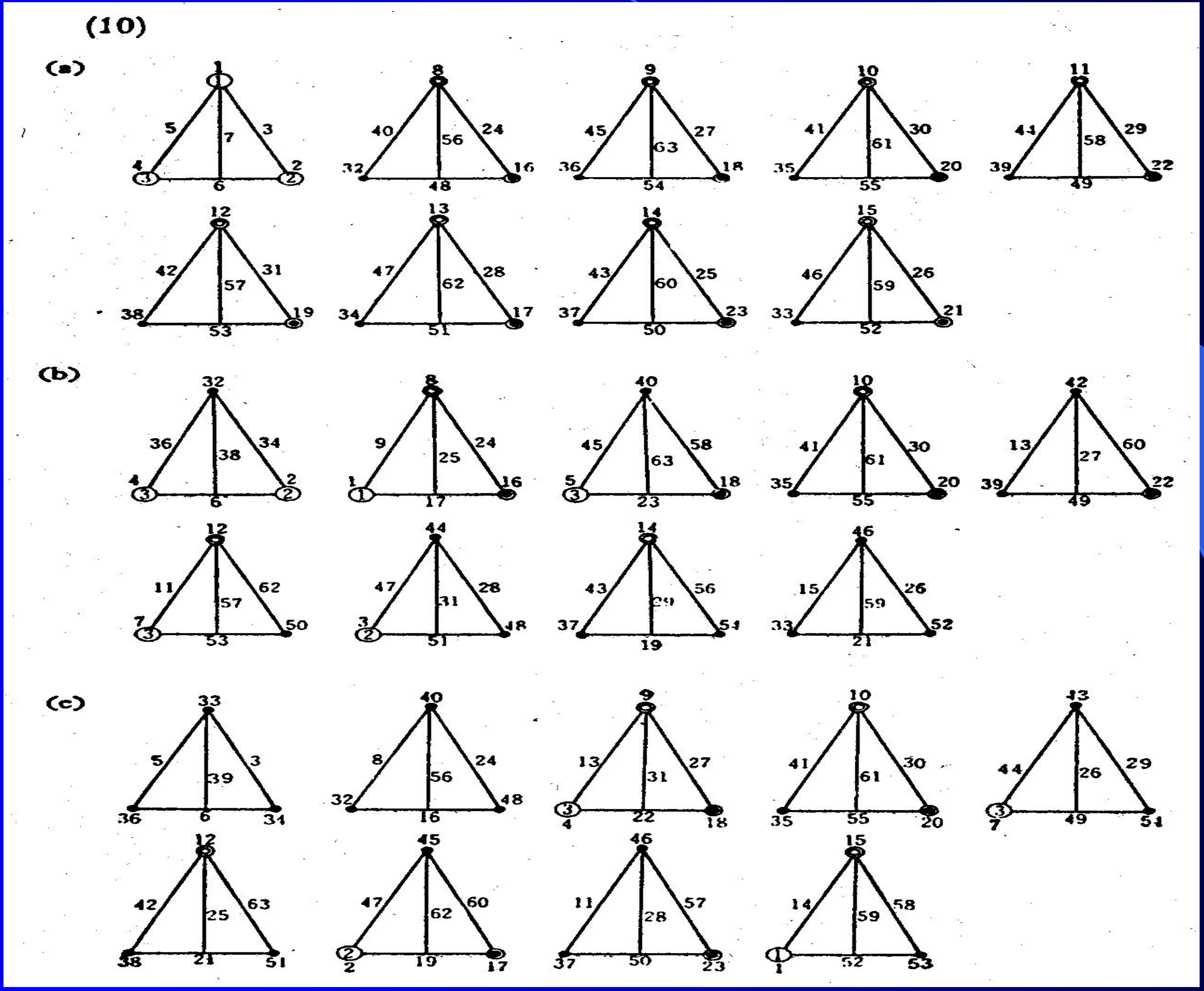


L_{12} (2^{11})

No.	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1
Group	1	2									

The L_{12} (2^{11}) is a specially designed array, in that interactions are distributed more or less uniformly to all columns. Note that there is no linear graph for this array. It should not be used to analyze interactions. The advantage of this design is its capability to investigate 11 main effects, making it a highly recommended array.

$L_{64}(2^{63})cont'd$



$L_9\ (3^4)$

No.	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1
	a	b	a b	a b^2
Group	1	2		

(1)

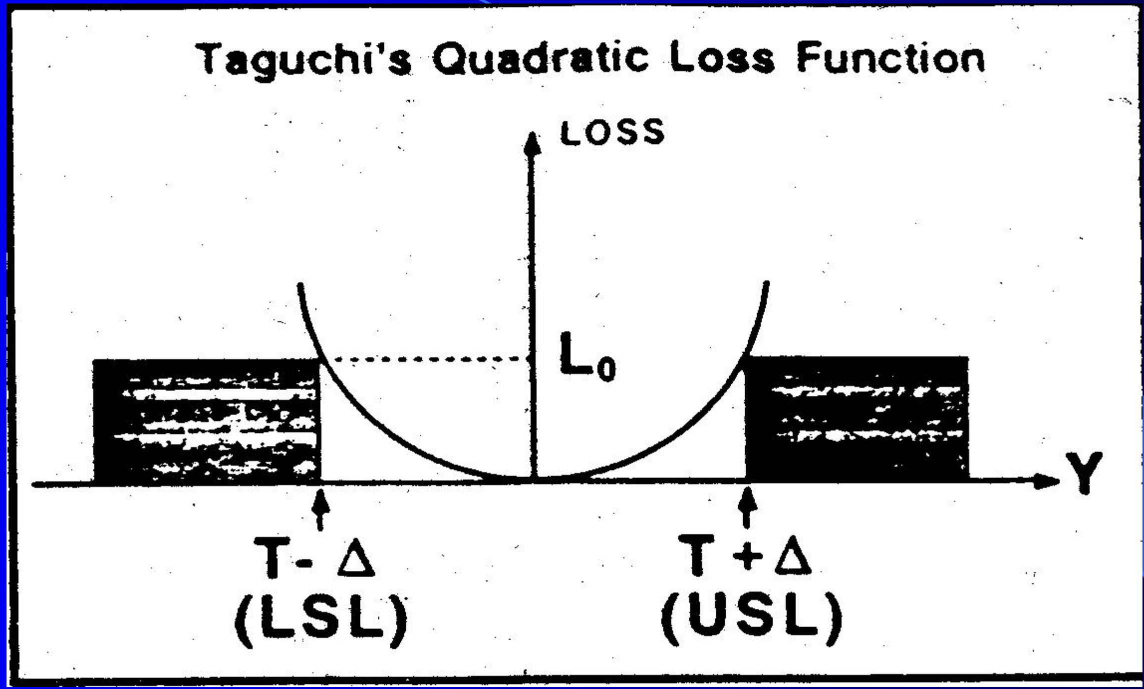
$L_{18}\ (2^1 \times 3^7)$

No.	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1
Group	1	2	3					

(1)

Note: Like the $L_{12}\ (2^{11})$, this is a specially designed array. An interaction is built in between the first two columns. This interaction information can be obtained without sacrificing any other column. Interactions between three-level columns are distributed more or less uniformly to all the other three-level columns, which permits investigation of main effects. Thus, it is a highly recommended array for experiments.

LOSS FUNCTION



$$L_i = k(y_i - T)^2$$

where
- y_i is the quality characteristic of interest for product i
- T is the quality characteristic target
- k is a constant that converts deviation to a monetary value

AVERAGE LOSS FOR n PRODUCTS

$$\overline{L} = k \sum (1/n)(y_i - T)^2$$

$$\text{where} \begin{cases} \text{n} = \text{sample size} \\ \text{y} = \text{value of the critical parameter} \\ \text{T} = \text{target value} \end{cases}$$

It may be shown that:

$$\textit{Average}\ \text{Loss} = \text{k} \bullet \{\text{Variance} + (\text{Off - Target Distance})^2\}$$

SIGNAL TO NOISE

- A logarithmic transformation of experimental data which considers both the mean and variability in an effort to reduce loss
  - Small is Better

$$S/N = -10 \log_{10} \frac{1}{n}\sum_{i=1}^{n}(Y_i^2)$$

  - Nominal is Better

$$S/N_N = 10 \log_{10} \frac{1}{n}\left(\frac{S_m - V_e}{V_e}\right)$$

$$\text{where } S_m = \frac{(\sum Y_i)^2}{n} \text{ and } V_e = \frac{\sum Y_i^2 - \frac{(\sum Y_i)^2}{n}}{n-1}$$

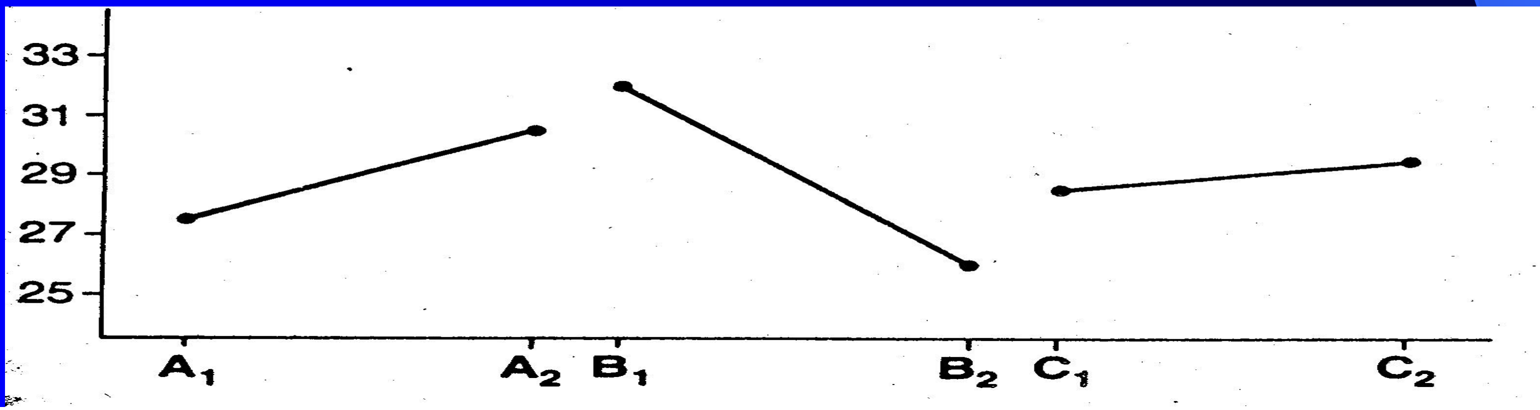
  - Larger is Better

$$S/N_L = -10 \log_{10} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{Y_i^2}\right)$$

Modeled Plastic Part Experiment

Factor	Level 1	Level 2
A Injection Pressure	205 psi	350 psi
B Mold Temperature	$150^\circ F$	$200^\circ F$
C Set Time	6 sec.	9 sec.

Condition	A B C	Results	A_1	A_2	B_1	B_2	C_1	C_2
1	1 1 1	30	30		30		30	
2	1 2 2	25	25			25		25
3	2 1 2	34		34	34			34
4	2 2 1	27		27		27	27	
		Total	55	61	64	52	57	59
		Average	27.5	30.5	32	26	28.5	29.5

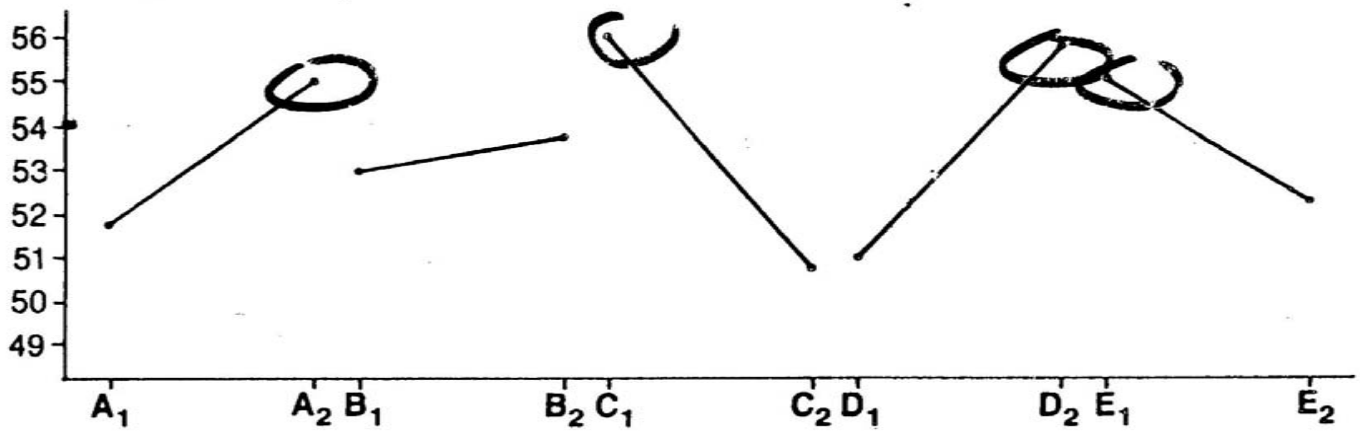


Sewn Seam Experiment

Factor		Level 1	Level 2
A	Tension	.5	1.0
B	Stitch length	10	12
C	Thread	#4	#6
D	Stitch type	straight	zigzag
E	Pressure	normal	high

Condition	A	B	C	D	E	Results	A_1	A_2	B_1	B_2	C_1	C_2	D_1	D_2	E_1	E_2
1	1	1	1	1	1	50	50		50		50		50		50	
2	1	1	1	2	2	58	58		58		58			58		58
3	1	2	2	1	1	52	52			52		52	52		52	
4	1	2	2	2	2	47	47			47		47		47		47
5	2	1	2	1	2	45		45	45			45	45			45
6	2	1	2	2	1	59		59	59			59		59	59	
7	2	2	1	1	2	57		57		57	57		57			57
8	2	2	1	2	1	59		59		59	59			59	59	
						Total	207	220	212	215	224	203	204	223	220	207
						Average	51.75	55	53	53.75	56	50.75	51	55.75	55	51.75

$A_2 C_1 D_2 E_1$



ADVANTAGES OF TAGUCHI METHODS

- Loss function
- Simplicity in selecting a design matrix
- Parameter design strategy for making products robust to noise
- Designs quality into the products as opposed to inspecting it out
- Thousands of success stories have been compiled through the American Supplier Institute

DISADVANTAGES OF TAGUCHI METHODS

- Simplicity in selecting a design matrix
- Poor modeling
- Using only signal to noise ratios, S/N_S, S/N_N, and S/N_L to identify dispersion
- Need for replication to identify dispersion effects
- De-emphasis of modeling interactions
- Some analysis techniques are unnecessarily complex
- Not providing guidance to experimenters on how to recover from unsuccessful experiments

田口式實驗設計之系統流程圖
(A systematic Problem Solving Flowchart for Taguchi Methods)

STAGE 1

Define the scope of the problem , State the objective of the experiment ; Brainstorm and Select numbers and levels for controllable and noise factors

STAGE2

Build an orthogonal design (Inner and outer Array)
$L_{12}(2^{11}), L_{18}(2 \times 3^{7})$ and $L_{36}(2^{3} \times 3^{13})$ designs are recently suggested by G. Taguchi. Determine the replications for each run.

STAGE 3

Run the experiment and collect the data , Then , a graphical analysis is conducted and the S/N Ratio is used . Important effects are determined to select a " optimal condition " or the " experimental champion " based on the best y (mean) or largest S/N

STAGE 4

Generate the Prediction equation for S/N ratio ; Conduct Confirmatory runs Compare the results Versus the prediction. Taguchi's Loss Function can be another index to assess the performance of " optimal condition".

The factors in the noise array are selected as well. Because several different types of assemblies are run through this wave solder process, two different types of assemblies were used. The objective is to find one setting for the wave solder process that is suitable for both types of assemblies. The design will also indicate if assembly type interacts with any of the controllable. In addition to product noise, both the conveyor speed and solder pot temperature will be moved around the initial setting given by the controllable array. This is because it is difficult to set the conveyor speed with any degree of accuracy and it is also difficult to maintain solder pot temperature. So the team of engineers chose to include these variables in the noise array variables to determine how much noise affects the process.

Table 1

A Designed Experiment for Wave Solder

An Example of the Use of Orthogonal Arrays

Controllable

Factors	Levels Low	Levels High
(1) Solder Pot Temperature (S)	480° F	510° F
(2) Conveyor Speed (C)	7.2 ft/m	10 ft/m
(3) Flux Density (F)	.9 °	1.0 °
(4) Preheat Temperature (P)	150 F	200 F
(5) Wave Height	0.5"	0.6"

Noise

Factors	
(1) Product Noise	Assembly #1, Assembly #2
(2) Conveyor Speed Tolerance	-0.2, +0.2 ft/m
(3) Solder Pot Tolerance	-5° F, +5° F

田口之配置/直交表—Orthogonal Arrays

Eight runs will be used to test effects of the five controllable in a Taguchi L_8 design (see table 2a). Notice that for each factor, there are four runs with the factor set at the high setting. This balancing is a property of the orthogonality of the set of runs. Table 2b lists the array of noise factors to be run at each of the eight setting of the controllable. This is a Taguchi L_4 design. The combination of the inner and outer arrays results in each urn of the controllable being repeated over the 4 combinations of the noise factor.

Table 2a

A Designed Experiment for Wave Solder

An Example of the Use of Orthogonal Arrays

Controllables Design
Inner Array

Run	Solder Pot Temperature	Conveyor Speed	Flux Density	Preheat Temperature	Wave Height
1	510	10.0	1.0	150	0.5
2	510	10.0	0.9	200	0.6
3	510	7.2	1.0	150	0.6
4	510	7.2	0.9	200	0.5
5	480	10.0	1.0	200	0.5
6	480	10.0	0.9	150	0.6
7	480	7.2	1.0	200	0.6
8	480	7.2	0.9	150	0.5

Table 2b

Outer Array

At each combination of the inner array, an outer array of noise factors is run.

RUN 1	RUN 2	RUN 3	RUN 4	Parameter
Assm#1	Assm#1	Assm#2	Assm#2	Product Noise
- 0.2	+ 0.2	- 0.2	+ 0.2	Conveyor Tolerance
- 5	+ 5	+ 5	- 5	Solder Tolerance

table 3

A Designed Experiment for Wave Solder

Combined Inner and Outer Arrays
Results

Noise Factors	RUN 1	2	3	4
Product Noise	#1	#1	#2	#2
Conveyor Tolerance	+0.2	+0.2	-0.2	+0.2
Solder Tolerance	- 5	+ 5	+ 5	- 5

Controllable Factors

Run	Solder	Conveyor	Flux	Preheat	Wave	1	2	3	4	Mean	S/N*	
1	510	10.0	1.0	150	0.5	194	197	193	275	215	-46.75	
2	510	10.0	0.9	200	0.6	136	136	132	136	135	-42.61	
3	510	7.2	1.0	150	0.6	185	261	264	264	244	-47.81	
4	510	7.2	0.9	200	0.5	47	125	127	42	85	-39.51	*
5	480	10.0	1.0	200	0.5	295	216	204	293	252	-48.15	
6	480	10.0	0.9	150	0.6	234	159	231	157	195	-45.97	
7	480	7.2	1.0	200	0.6	328	326	247	322	305	-49.76	
8	480	7.2	0.9	150	0.5	186	187	105	104	145	-43.59	

* EXPERIMENTAL CHAMPION

table 4

A Designed Experiment for Wave Solder

Analysis

Parameter	Level	Mean	S/N	
Solder Pot Temperature	480	225	-46.87	
	510	170	-44.17	*
Conveyor Speed	7.2	195	-45.17	
	10.0	200	-45.87	
Flux Density	0.9	140	-42.91	*
	1.0	255	-48.11	
Preheat Temperature	150	200	-46.03	
	200	194	-45.01	
Wave Height	0.5"	174	-44.50	
	0.6"	220	-46.54	
Interaction		200	-45.68	
		194	-45.36	

* These are the optimum level settings for each factor based on S/N. Factors without an asterisk are not significant and their levels can be based on other considerations.

figure 5

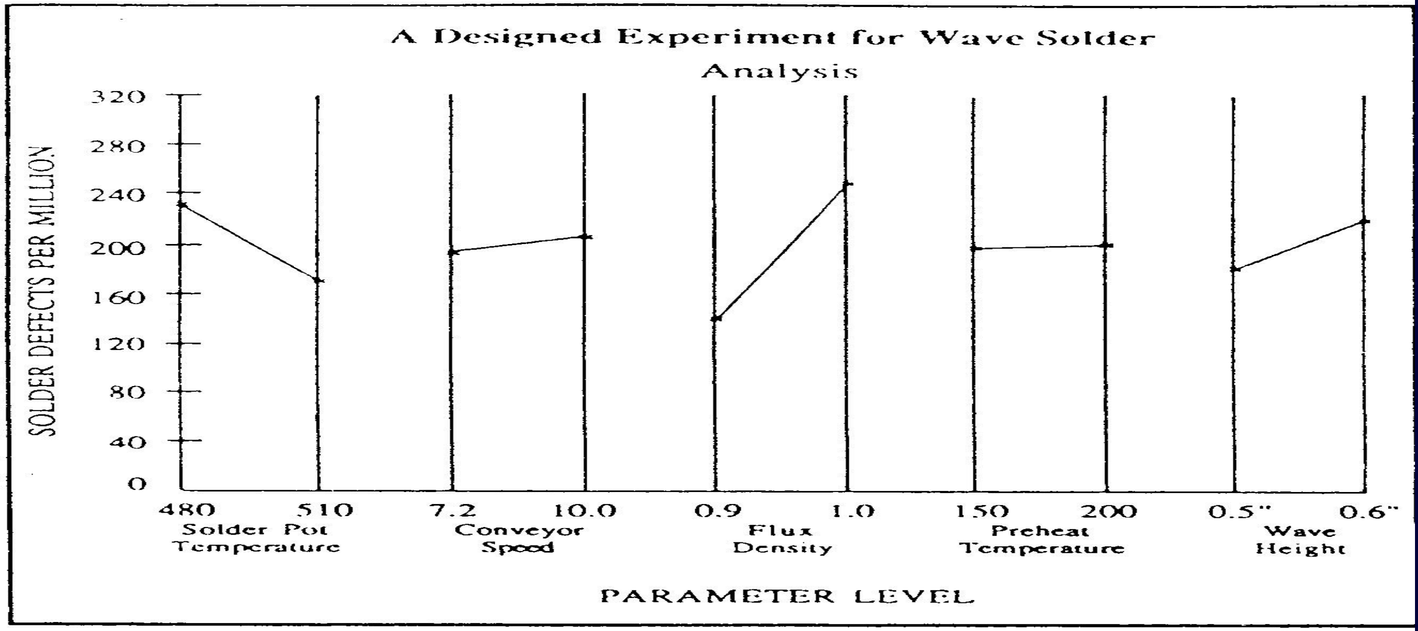
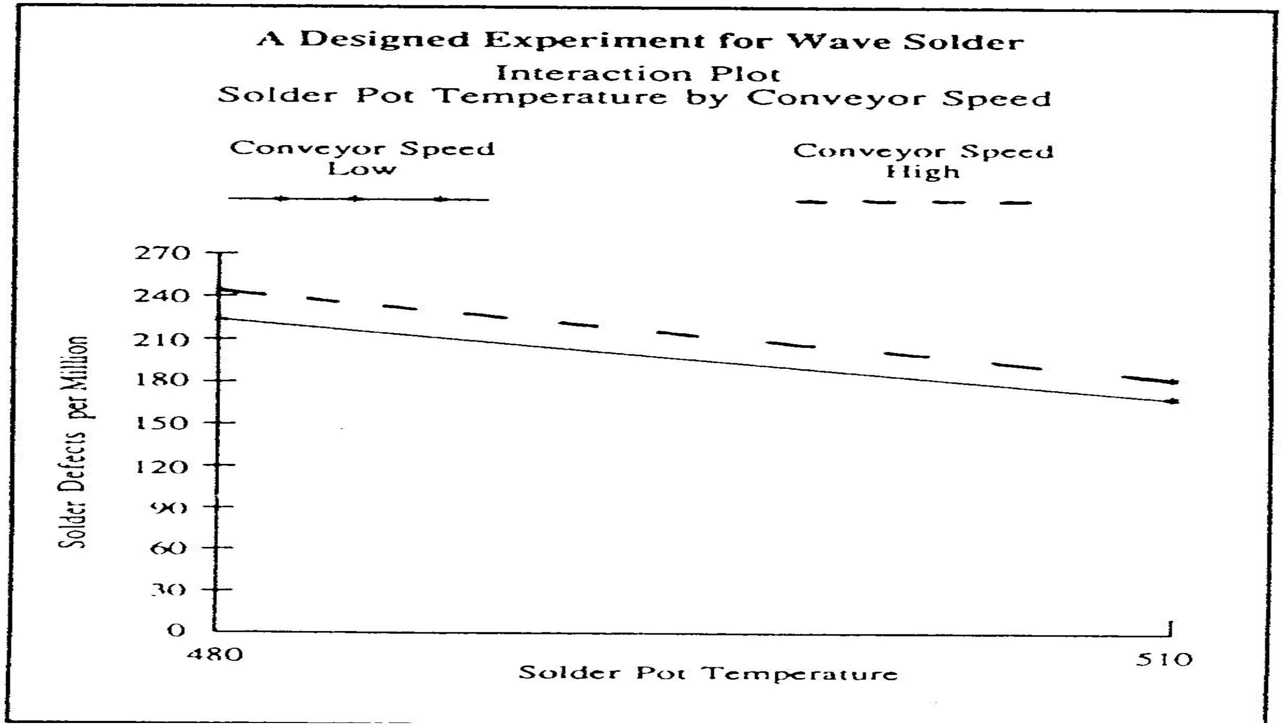


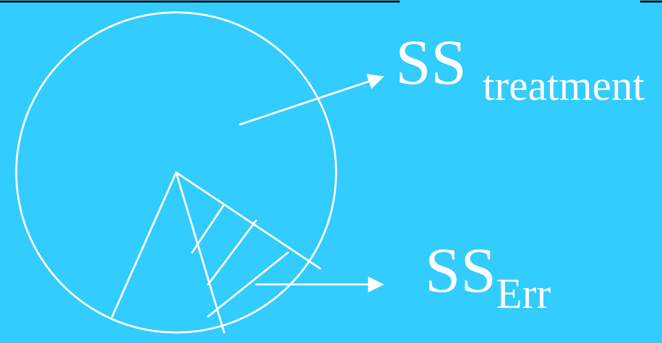
figure 6



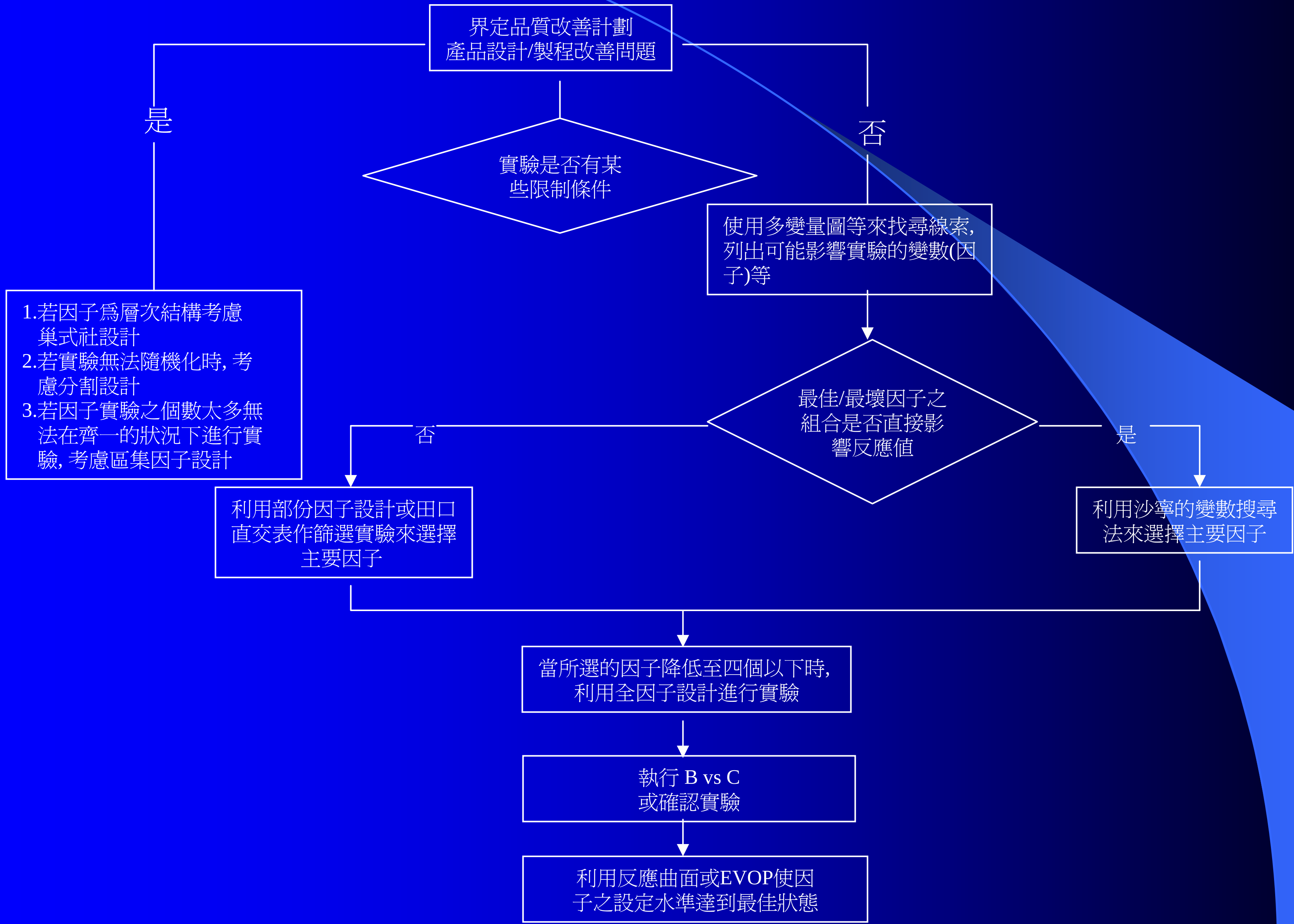
實驗之選擇

實驗設計別	適用條件	其他特徵
(1)2^{k-2} or Ⅲ (田口直交表)	對製程了解少，變數多(因子個數多)且不包含在實驗內之其他變數(因子)可固定。實驗可以任何順序進行，不受限制。進行實驗耗時且成本高。	無法測出一些非線性之影響(可利用中心複合設計增加軸點)，須與"鏡射實驗"聯合應用方能分離出主因子與交互作用之影響。
(2) 2^{k-1}, ⅣorⅤ (解析度)	交互作用之可能性高；但實際因子組合之情況未知。與(1)比其實驗成本較低。	Ⅴ：在三階交互作用可忽略之情形下，主因子與二階交互作用均可被分離出 Ⅳ：主因子可分離出，但二階之交互作用則無法。

實驗之選擇

實驗設計別	適用條件	其他特徵
(3)全因子設計	只有少數之因子須試驗(k≦4)，三階以上複雜之交互作用極可能發生	超過四個因子以上，實驗不但耗時且昂貴。一般而言，因子之水準數亦只考慮二個或三個水準。
(4) 區集設計	實驗結果受時間、材料、環境之影響而產生變異 / 誤差。為控制實驗之誤差，須予以區隔。	 $F = MS_{trt} / MS_{Err}$ $F < F_{\alpha}$

選擇適當實驗設計方法之決策流程圖



沙寧實驗設計七大手法

方法	目的	適用性	使用時機	樣本大小
多變量圖	1. 發覺 Red X 出現的位置,週期或時間 2. 偵測出非隨機變異的變化模式	在不同時間,經由連續分層抽取之樣本爲可量度之情形下	1. 用於工程上產品雛形產製測試實驗 2. 解決生產問題	>=9
元件搜尋	從眾多影響品質之變數中找 RedX	當好及壞的裝配元件可分解並重新組裝時	1. 用於當只有少數樣本可取得之產品雛形產製階段或測事實驗 2. 解決生產問題	=2
配對比較	與元件搜尋相同	當好及壞的裝配元件不可分解時	主要用於生產,退貨或故障分析	>=12
變數搜尋	1. 找出 RedX, Pink X 等 2. 分離及量化變數的主效果和交互作用效果	在多變量圖,元件搜尋與配對比較後,集中焦點在搜尋主要因子	1. 當影響品質之變數爲 4 個以下 2. 在雛形產製測試階段 3. 解決生產問題	5 個變數時 6-16, 超過 5 個變數時，每多一個變數多作 2 次
全因子實驗	與變數搜尋相同	與變數搜尋相同	1. 當影響品質之變數爲 4 個以下 2. 與變數搜尋相同	最多 16 或 32 次
B 與 C 法比較	1. 評估並確認 B 法與 C 法何者爲佳之方式 2. 假如實行結果沒有不同但成本不同,選擇 B 或 C	1. 確認在變數搜尋或全因子實驗中找出來的重要因子 2.當問題易解只用作實驗而可忽視其他的方法時	1. 在雛形產製測試階段 2. 在生產階段,求更加品質時 3. 可視爲一般性的方法,用於非技術性的領域,如銷售,廣告等	通常爲 3 次 B,3 次 C
散佈圖	1. 決定重要變數的最佳值 2. 降低不重要變數的成本	在前述 6 個方法使用之後	產品之雛形產製測試階段	30 次