New Process Capability Indices with Robust Estimators for Autocorrelated Data

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Abstract
Traditionally, the process capability index is developed assuming that the process output data are independent and follow normal distribution. However, in most environmental cases, the process data are autocorrelated. The autocorrelated process, if unrecognized as an independent process, can lead to erroneous decision-making and unnecessary quality loss. In this paper, three robust estimators of capability indices are proposed to relieve the independence assumption for nominal-the-best and smaller-the-better cases. Furthermore, we use mean squared error and mean absolute percent error to compare the accuracy of our proposed indices to non-independent indices. The results show that our proposed capability indices are more accurate.

Keywords: Process capability index, Autocorrelated process, Mean absolute percent error

1. Introduction
Process capability analysis is a very important SPC tool for monitoring and evaluating process performance. Traditionally, the process capability index is developed assuming that the process output data are independent and follow normal distribution. However, the quality characteristics of many manufacturing processes in chemical, pharmaceutical and electronic industries often exhibit the property of autocorrelation. The autocorrelated process, if unrecognized as an independent process, can lead to erroneous decision-making and unnecessary quality loss. Therefore, the purpose of this research is to develop new process capability indices with robust estimators to relieve the independence assumption for two common engineering specifications, i.e. the-nominal-the-best and the-smaller-the-better cases.

Using the model-free concept proposed by (Shore, 1997) and by modifying the autocorrelated process capability indices proposed by (Zhang, 1998), (Scaglìarini, 2002), (Sun et al., 2010), we develop two new autocorrelation process capability indices.

In addition, though process capability indices have been widely applied in evaluating the quality performance of manufacturing processes in the past decade, they are seldom applied to the evaluation of environmental performance, especially for dependent or autocorrelated data.

Fast economic development over the past 50 years has caused serious environmental problems that have reduced the quality of life for people world-wide. To reduce environmental contamination such as air pollution, major enterprises and corporations around the world have begun to systematically review environmental performance based on regulations stipulated by Environmental Protection Agency in each country. To prevent further environmental deterioration, the establishment of risk indices to continuously monitor and evaluate
environmental impacts has become an important aspect of environmental research. Since autocorrelated data exist in most environmental processes, autocorrelation adjustment should be considered in estimating process capability indices to monitor and evaluate environmental performance.

2. Literature Review

2.1. Process capability indices for autocorrelated data

Suppose the quality characteristic of a manufacturing process follows a first-order stationary autoregressive AR(1) process with parameter \( \phi \) such that \(|\phi| < 1\). Then the process can be written as

\[
X_t - \mu = \phi (X_{t-1} - \mu) + a_t
\]  

(1)

where \( a_t \) is a white noise with zero mean and variance \( \sigma_a^2 \), \( a_t \sim N(0, \sigma_a^2) \) and \( \mu \) is the mean of process. In this case, the variance of the process is \( \text{Var}(X_t) = \sigma_r^2/(1-\phi^2) \).

(Wallgren, 1996) studied \( C_{pm} \) index for autocorrelated data and proposed \( C_{pmr} \) index as

\[
C_{pmr} = \frac{USL - LSL}{6\sqrt{\sigma_r^2 + (\mu - T)^2}}.
\]  

(2)

where \( \sigma_r^2 = \sigma_a^2/(1-\phi^2) \). (Wallgren, 2001) further extended \( C_{ph} \) index for autocorrelated data and proposed

\[
C_{pk} = \min\left(\frac{USL - \mu}{3\sigma_r}, \frac{\mu - LSL}{3\sigma_r}\right).
\]  

(3)

(Zhang, 1998) studied the indices \( C_p \) and \( C_{ph} \) for autocorrelated data. (Guevara and Vargas, 2007) extended his study to the indices \( C_{pm} \) and \( C_{pmk} \) and made a general comparison of these four indices. (Scagliarini, 2002) investigated the behavior of the estimator of \( C_p \) in the case of autocorrelated data. They derived the expected value of estimator \( \hat{C}_p \) as follows

\[
E(\hat{C}_p) \cong C_p \left[ \frac{1}{f(n, \rho_i)} \right]^{1/2} \left[ 1 + \frac{3F(n, \rho_i)}{4(n-1)^2[f(n, \rho)^2]} \right],
\]  

(4)

where \( f(n, \rho_i) = 1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} (n-i) \rho_i \),

\[
F(n, \rho) = n + 2 \sum_{i=1}^{n} (n-i) \rho_i + \frac{1}{n^2} \left[ n + 2 \sum_{i=1}^{n} (n-i) \rho_i \right]^{-2} - \frac{2}{n^2} \sum_{i=1}^{n} \sum_{j=0}^{n-i} (n-i-j) \rho_i \rho_j
\]

and \( \rho_i \) is the autocorrelation coefficient at lag \( i \). (Sun et al., 2010) investigated five estimation schemes of process capability with autocorrelated data. They showed the expected value of estimator \( \hat{C}_{pm} \) can be expressed as

\[
E(\hat{C}_{pm}) \cong C_{pm} \left[ 1 + \frac{3G(n, k_2, \rho_i)}{4n(1+k_2)^2} \right],
\]  

(5)
where \( G(n, k, \rho) = 1 + \frac{2}{n} \sum_{i=1}^{n-1} (n-i) \rho_i^2 + 2k_2 \left( 1 + \frac{2}{n} \sum_{i=1}^{n-1} \rho_i \right) \).

(Sun et al., 2010) defined the unbiased estimators of \( C_p \) and \( C_{pm} \) indices as

\[
\hat{C}_p = \frac{\hat{C}_p}{B(\hat{C}_p, n, \hat{\rho}_1^i) + 1}, \quad \hat{C}_{pm} = \frac{\hat{C}_{pm}}{B(\hat{C}_{pm}, n, \hat{\rho}_1^i, k_2) + 1},
\]

where

\[
B(\hat{C}_p, n, \hat{\rho}_1^i) = \frac{1}{f(n, \hat{\rho}_1^i)^2} \left( 1 + \frac{3F(n, \hat{\rho}_1^i)}{4(n-1)^2 f(n, \hat{\rho}_1^i)} \right) - 1,
\]

\[
B(\hat{C}_{pm}, n, \hat{\rho}_1^i, k_2) = \left( 1 + \frac{3G(n, k_2, \hat{\rho}_1^i)}{4n(1+k_2)^2} \right) - 1,
\]

where \( \hat{\rho}_1^i \) is the estimated autocorrelation coefficient in the AR (1) model.

3. Modifying the Unbiased Estimators of Capability Indices for Autocorrelated Data

3.1. The-nominal-the-best case

Suppose \( \{X_1, X_2, ..., X_n\} \) are taken from the AR(1) process. The sample mean and sample variance can be calculated as \( \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \) and \( S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \). The autocorrelation coefficients at lag i can be estimated as

\[
\hat{\rho}_i = \frac{1}{n} \sum_{t=i}^{n} (X_t - \bar{X})(X_{t+i} - \bar{X}) \sum_{t=i}^{n} (X_t - \bar{X})^2 . \tag{7}
\]

Because (Box and Jenkins, 1976) showed that the sample autocorrelation coefficients \( \hat{\rho}_i \) are more reliable when the lag period is \( i \leq n/4 \), we calculate the first \([n/4]\) sample autocorrelation coefficients as \( \{\hat{\rho}_1, \hat{\rho}_2, ..., \hat{\rho}_{[n/4]}\} \). Note that \([\ ]\) denotes the Gaussian symbol.

Then we use the following statistics proposed by (Ljung-Box, 1978) to test the significance of autocorrelation coefficients:

\[
Q_{LB}(p) = n(n+2) \sum_{i=1}^{p} \hat{\rho}_i^2 / (n-i). \tag{1}
\]

When the autocorrelation coefficient at lag \( k \) is not significant, we set \( \hat{\rho}_j = 0, \forall j \geq k \).

Instead of using all lags of the sample autocorrelation coefficients, we calculate the first \([n/4]\) sample autocorrelation coefficients \( \{\hat{\rho}_1, \hat{\rho}_2, ..., \hat{\rho}_{[n/4]}\} \) and propose that the modified unbiased estimators of \( C_p \) and \( C_{pm} \) for the autocorrelated data are:

\[
N\hat{C}_{pu} = \frac{\hat{C}_p}{b(n, \hat{\rho})}, \quad N\hat{C}_{pm} = \frac{\hat{C}_{pm}}{b(n, \hat{\rho}_1^i, k_2)}, \tag{9}
\]
where \( \hat{C}_p = \frac{USL - LSL}{6S} \), \( \hat{C}_{pu} = \frac{USL - LSL}{6\sqrt{S^2 + (\bar{X} - T)^2}} \).

\[
b(n, \hat{\rho}_i) = \frac{1}{\left[f(n, \hat{\rho}_i)\right]^{1/2}} \left[1 + \frac{3F(n, \hat{\rho}_i)}{4(n-1)^2 \left[f(n, \hat{\rho}_i)\right]^2}\right] b(n, \hat{\rho}_i, k_z) = 1 + \frac{3G(n, k_z, \hat{\rho}_i)}{4n(1 + k_z)^2},
\]

\[
f(n, \hat{\rho}_i) = 1 - \frac{2}{n(n-1)} \sum_{i=1}^{\frac{n\lceil \frac{n}{4} \rceil}{4}} (n-i) \hat{\rho}_i \quad (10)
\]

\[
F(n, \hat{\rho}_i) = n + 2 \sum_{i=1}^{\frac{n\lceil \frac{n}{4} \rceil}{4}} (n-i) \hat{\rho}_i^2 + \frac{1}{n^2} \left[n + 2 \sum_{i=1}^{\frac{n\lceil \frac{n}{4} \rceil}{4}} (n-i) \hat{\rho}_i \right]^2 - \frac{2}{n} \sum_{i=0}^{\frac{n\lceil \frac{n}{4} \rceil}{4}} \sum_{j=0}^{n-i} (n-i-j) \hat{\rho}_i \hat{\rho}_j \quad (11)
\]

\[
G(n, k_z, \hat{\rho}_i) = 1 + \frac{2}{n} \sum_{i=1}^{\frac{n\lceil \frac{n}{4} \rceil}{4}} (n-i) \hat{\rho}_i^2 + 2k_z \left(1 + \frac{2}{n} \sum_{i=1}^{\frac{n\lceil \frac{n}{4} \rceil}{4}} \hat{\rho}_i \right) k_z = (\bar{X} - T) / S.
\]

### 3.2. The-smaller-the-better case

For the-smaller-the-better case, we extend \( C_{pu} \) index proposed by (Kane, 1986) for autocorrelated data. The estimator of \( C_{pu} \) index can be written as

\[
\hat{C}_{pu} = \frac{USL - \bar{X}}{3S},
\]

where \( S \) is the sample standard deviation. According to the results of (Zhang, 1998), the correlation coefficient between the sample mean \( \bar{X} \) and the sample standard deviation \( S \) approaches zero. Thus, we have

\[
E(\hat{C}_{pu}) = E\left(\frac{USL - \bar{X}}{3S}\right) = \frac{USL - E(\mu)}{3} E\left(\frac{1}{S}\right).
\]

(13)

Using the expected value of the inverse of the sample standard deviation proposed by (Scagliarini, 2002), the expected value of \( \hat{C}_{pu} \) can be derived as

\[
E(\hat{C}_{pu}) = \frac{USL - E(\mu)}{3} E\left(\frac{1}{S}\right) \equiv \frac{USL - \mu}{3} \cdot \frac{1}{\sigma \left[f(n, \hat{\rho}_i)\right]^{1/2}} \left[1 + \frac{3F(n, \hat{\rho}_i)}{4(n-1)^2 \left[f(n, \hat{\rho}_i)\right]^2}\right]
\]

(14)

where \( f(n, \hat{\rho}_i) \) and \( F(n, \hat{\rho}_i) \) are defined in (10) and (11). Then, the modified unbiased estimator of \( C_{pu} \) index for autocorrelated data is defined as

\[
N\hat{C}_{pu} = \frac{\hat{C}_{pu}}{b(n, \hat{\rho}_i)}, \quad \text{where } b(n, \hat{\rho}_i) = \frac{1}{\left[f(n, \hat{\rho}_i)\right]^{1/2}} \left[1 + \frac{3F(n, \hat{\rho}_i)}{4(n-1)^2 \left[f(n, \hat{\rho}_i)\right]^2}\right].
\]

(15)
4. Simulation results and analysis

4.1. Comparison of the average PCI values

To compare the PCIs performance under different time series models, we use the average PCI values from the simulation results. The time series model AR(1), MA(1) below have been used for comparison.

\[
\text{AR(1)} \quad X_t = \phi_1 X_{t-1} + a_t \tag{16}
\]

\[
\text{MA(1)} \quad X_t = \mu + a_t - \theta_1 a_{t-1} \tag{17}
\]

The parameter \(\phi_1\) or \(\theta_1\) is assumed to be -0.9, -0.7, ..., 0.7 or 0.9. We set the variance of white noise \(\sigma_a^2 = 1\), process mean \(\mu = 0\) and target value \(T = 0\). In addition, the specification limits are set such that the nonconforming rate \(p\) is equal to 0.27%. With these settings, the true values of \(C_p\) and \(C_{pa}\) indices are equal to 1 and the true value of \(C_{pa}\) index is equal to 0.927. Under AR(1) or MA(1) model, the average PCI values are computed based on 5,000 random samplings for \(n=200\). The simulation results are shown in Figure 1. In Figure 1(a), we found that the average values of \(\hat{C}_p\) or \(\hat{C}_{pr}\) index are deviated from the true value 1. In contract, the averages of \(\hat{C}_{pa}\) or \(\hat{C}_{pr}\) index are more close to the true value 1 except \(\phi_1 = -0.9\) or 0.9. In Figure 1(b), we found that the average values of \(\hat{C}_p\) or \(\hat{C}_{pr}\) index are deviated from the true value 1. The average value of \(\hat{C}_{pa}\) index is deviated from the true value except for the case of \(\theta_1 = 0\). The average values of \(\hat{N\hat{C}}_pa\) index are close to true value under MA(1) model. Based on the simulation results shown in Figures 1, we can conclude that our proposed \(\hat{N\hat{C}}_{pa}\) index outperforms the existing PCIs under AR(1) or MA(1) model.

![Figure 1](image1.png)

*Figure 1-Comparison of various \(C_p\) indices under (a) AR(1) and (b) MA(1) model with different parameters \(\phi_1\) and \(\theta_1\)*

In Figures 2, the average value of the \(\hat{N\hat{C}}_{pa}\) index was closer to its true value than the other indices. Based on the simulation results shown in Figures 2, we conclude that our proposed \(\hat{N\hat{C}}_{pa}\) and \(\hat{N\hat{C}}_{pa}\) indices outperform existing PCIs under both AR(1) and MA(1).


**Figure 2** - Comparison of various $C_{pu}$ indices under (a) AR(1) and (b) MA(1) model with different parameters $\phi$ and $\theta$.

### 4.2. Assessing the accuracy of various process capacity indices

To evaluate the performance of various process capability indices (PCI), we use the following mean squared error (MSE) and mean absolute percent error (MAPE) as the criteria for comparison. The MSE and the MAPE as shown below can be referred to (Render et al., 2009).

\[
MSE = \frac{1}{n} \sum_{i=1}^{n} (\hat{\theta}_i - \theta_i)^2, \quad (18)
\]

\[
MAPE = \left( \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\hat{\theta}_i - \theta_i}{\theta_i} \right| \right) \cdot 100\%, \quad (19)
\]

where $\theta_i$ represents the true PCI value, $\hat{\theta}_i$ represents the estimated PCI value.

Meanwhile, two-stationary time series models AR(1) and ARMA(2,1) are considered for assessing the accuracy of various PCIs. In order to compare the PCIs’ performance under various parameter combinations, we first calculate their MSE and MAPE. As shown in Equation 14 and 15, MSE and MAPE are two commonly used relative errors. Then, we utilize the Rank Sum of Relative Errors (RSRE) proposed by (Pan and Chen, 2010) to evaluate the accuracy of various process capability indices. To further show that our proposed $NC_{pu}$ index has the best overall performance in terms of accuracy in the nominal-the-best case, comparison of RSRE values for various $C_p$ indices under various combinations of time series models and nonconforming rates is listed in Appendix A. On the other hand, we show that our proposed $NC_{pu}$ index has the best overall performance in terms of accuracy and robustness in the nominal-the-best case. Moreover, comparison of the RSRE values for various $C_{pu}$ indices under different combinations of time series models and nonconforming rates are listed in Appendix B, where we show that our proposed $NC_{pu}$ index has the best overall performance in terms of accuracy and robustness for the smaller-the-better case.

In practical applications, the process capability index proposed by (Wallgren, 1996; 2001) requires proper model fitting and hence increases the difficulty of calculation. (Sun et al., 2010) considers that process data only follow the AR(1) model in deriving process capability indices. On the other hand, we have taken two different time series models into consideration. The simulation results show that our capability indices are more robust than the previous PCIs by eliminating the insignificant values from the first $\lfloor n/4 \rfloor$ of sample autocorrelation coefficients.
In the past 50 years, the government of Taiwan has implemented several rapid, aggressive industrial development projects to create an astonishing economic miracle. These projects range from the construction of petro refineries and steel plants to plastic and petrochemical manufacturing facilities. Yet this fast economic development has also caused serious environmental contamination and air pollution in Taiwan. Carbon monoxide (CO) and nitrogen dioxide (NO₂) are two of the main pollutants created. These have serious effects not only on air quality, but on the increased susceptibility to respiratory disease in Taiwan. As a result, the Environmental Protection Agency (EPA) of Taiwan has enacted various air quality standards since July, 1992. The EPA standards of NO₂ (yearly average) and CO (eight-hour average) are 0.05 and 9 ppm (particle per million) respectively.

The numerical analysis for our CI study focuses on the NO₂ and CO emission data from 1993 to 2011. Hence, we use two time series for the annual NO₂ and CO emission data. In Figure 3, we found the yearly average data of NO₂ emissions is well below its specification limit of 50 ppb (i.e. 0.05ppm). Since the yearly average data was not available for CO emissions, we used the 8-hour average data. Figure 3 shows that the 8-hour average data for CO emissions is well below its specification limit of 9 ppm, as well. Because CO data did not follow a normal distribution, we performed the (Box-Cox, 1964) transformation to convert CO emission data into a normal distribution. The Shapiro test results indicate that the annual NO₂ and transformed CO emission data are normally distributed. The unit root test results further indicate that the annual NO₂ and transformed CO emission time series are stationary. Then, we used our autocorrelation capability index \( \hat{NC}_{pu} \) to evaluate the air quality and compare it to previous \( \hat{C}_{pu} \), \( \hat{C}_{pu} \), and \( \hat{C}_{pu} \) indices. We used \( \hat{NC}_{pu} \) in this case since air quality data belongs to the smaller-the-better case. The upper specification limits (USL) for NO₂ and CO are listed as the EPA standards.
(1) Process capability analysis for nitrogen dioxide (NO$_2$) emission data

The trend for NO$_2$ emissions is declining, and they are well below their USL. This suggests that the annual NO$_2$ emission over the past 19 years has been significantly reduced. Moreover, we compare our proposed $N\hat{C}_{puu}$ index = 1.894 with the following calculated capability indices: $\hat{C}_{pu}$ = 2.592, $\hat{C}_{pur}$ = 1.611, and $\hat{C}_{puu}$ = 0.760 based on AR (1) model fitting for the NO$_2$ emission data. Apparently, $\hat{C}_{pu}$ index is far too high and results in an overestimation since it does not take autocorrelation into account. The $\hat{C}_{puu}$ index, on the other hand, is significantly underestimated. Thus, our proposed $N\hat{C}_{puu}$ index can be considered the most reasonable one to reflect the actual performance of NO$_2$ emissions in Taiwan.

(2) Process capability analysis for carbon monoxide (CO) emission data

The trend for CO emissions is also declining, and they are far below their USL. This suggests that the annual CO emissions over the past 18 years have been well regulated. Moreover, we compare our proposed $N\hat{C}_{puu}$ index = 3.932 with the following calculated capability indices: $\hat{C}_{pu}$ = 6.030, $\hat{C}_{pur}$ = 4.673, and $\hat{C}_{puu}$ = 1.314 based on the AR (1) model fitting for the transformed CO emission data. Apparently, $\hat{C}_{pu}$ index is far too high and results in an overestimation since it does not take autocorrelation into account, while the $\hat{C}_{puu}$ index is too low and results in an underestimation. Thus, our proposed $N\hat{C}_{puu}$ index can be considered the most reasonable one to reflect the actual performance of CO emissions in Taiwan.

Ever since the Environmental Protection Agency released its air quality standards in July of 1992, corrective and preventive measures for reducing released pollutants has been undertaken by the manufacturing sector in Taiwan. Our analysis confirms that both the NO$_2$ and CO emissions level improved significantly.

6. Conclusions

This research has established process capability performance indices for evaluating the environmental impacts caused by air pollutants and other sources of contamination. Since most environmental data are autocorrelated, the traditional process capability indices based on an independence assumption can no longer be applied to the monitoring and evaluation of environmental performance. Hence, robust estimators for capability indices are proposed to relieve this assumption in the-nominal-the-best and the-smaller-the-better cases. Furthermore, we use MSE (mean squared error) and MAPE (mean absolute percent error) to compare the accuracies of our proposed capability indices with their predecessors. The results show that our proposed capability indices are more robust. Finally, we demonstrate that the new indices can be applied to the performance evaluation of NO$_2$ and CO emissions in Taiwan. The results and contribution of this research is summarized as follows:
1. Given an autocorrelated manufacturing process under the-nominal-the-best case, two new capability indices $NC_{pa}$, $NC_{pma}$ with robust estimators were proposed. The simulation results show that our proposed $NC_{pa}$, $NC_{pma}$ indices accurately reflect the actual process performance.

2. Given an autocorrelated manufacturing process under the-smaller-the-better case, a new capability index $NC_{pua}$ was proposed. The simulation results show that our proposed $NC_{pua}$ index accurately reflects the actual process performance.

3. In the numerical example, we use the two major air pollutants CO and NO$_2$ emission data of Taiwan to demonstrate that our proposed indices can be successfully applied in monitoring and evaluating air quality performance.

References


### Appendices

**A - Comparison of RSRE values for various $C_p$ indices under different combinations of time series models and nonconforming rates (the-nominal-the-best case)**

<table>
<thead>
<tr>
<th>Model</th>
<th>$p$</th>
<th>$\hat{C}_p$</th>
<th>$\hat{C}_{pr}$</th>
<th>$\hat{C}_{pa}$</th>
<th>$N\hat{C}_{pa}$</th>
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</thead>
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<tr>
<td>AR(1)</td>
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<td>1.6646</td>
<td>1.2841</td>
<td>1.0751</td>
<td>0.4728</td>
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<tr>
<td></td>
<td>0.0063%</td>
<td>2.9012</td>
<td>2.1762</td>
<td>1.8534</td>
<td>0.8152</td>
</tr>
<tr>
<td>ARMA(2,1)</td>
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<td>26.9523</td>
<td>6.345</td>
<td>5.6069</td>
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<td></td>
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<td>6.2566</td>
<td>17.7925</td>
<td>4.3319</td>
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<th>Overall rank sum of relative errors (RSRE using MAPE)</th>
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<td>Model</td>
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<tr>
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<tr>
<td>AR(1)</td>
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<td></td>
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<tr>
<td>ARMA(2,1)</td>
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**B - Comparison of RSRE values for various $C_{pu}$ indices under different combinations of time series models and nonconforming rates**

<table>
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<th>Model</th>
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<th>$\hat{C}_{pr}$</th>
<th>$\hat{C}_{pa}$</th>
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