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Evolving the latent variable model as an environmental DEA technology

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ABSTRACT

This article tests several nonparametric DEA models for their ability to accurately decompose CO_2 emissions change using a Malmquist styled decomposition framework. This production oriented activity analysis involves panel data and two data sets from the literature for comparison. A new Latent Variable radial input-oriented technology is introduced that is closely associated with a Koopmans Efficient Slacks Based Model. The Latent Variable technology simultaneously reduces inputs and undesirable outputs in a single Multiple Objective Linear Program. This production theoretic methodology is adapted to preserve both scale efficiency and causality within the envelopment framework. Finally, the application studies demonstrate the internal consistency of the Latent Variable reduction coefficients, which overturns previous results and paves the way for further research into undesirable externalities.

1. Reducing undesirable outputs

Driven by concern for the environment, a preponderance of articles addressing the reduction of undesirable outputs has been published in the last decade. In a 2008 survey done by Zhou et al. [1] 72 of 100 articles reviewed were written since 1999. If the output of undesirable greenhouse gases (GHG) is not drastically reduced in the following decade, tens of thousands of species could become extinct due to pollution, rapid climate change and global warming induced by GHG such as CO_2 [2]. The effects on human food security and other key vulnerabilities may also be devastating: increased poverty, and deepening economic and political strife. These and other 'externalities,' will need to be taken into account as 'undesirable outputs' are recognized to be more than just an environmental concern. This seems to be the direction that DEA is moving as a multiple input/output system that is not manacled by statistical distribution or invariance issues. Management scientists have yet to develop a consistent approach for the measurement of environmental performance using DEA let alone setting up a basis for its expanded use given other externalities. A variety of methods are currently used which impose restraints on outputs while often ignoring production assumptions or causal relationships. Hence, we propose the Latent Variable Environmental Model (LVM) to act as a basis for the consistent simultaneous reduction of inputs and undesirable outputs as a first step in extending the scope of Data Envelopment Analysis.

Charnes et al. in 1978 characterized the CCR Model [3] as the fundamental DEA reference technology in envelopment form for either an input or output orientation with Constant Returns To Scale, CRS. Its efficiency measure was radial-based using efficiency ratios rather than the additive form of the slacks-based approach that they (Charnes, Cooper with others) would also pioneer in 1985, SBM [4]. The Slacks Based Model is Pareto–Koopmans [5,6] efficient since it removes undetected slacks from the radial models, usually in a two-stage process. DMUs on the efficiency frontier using radial efficiency measures are Farrell efficient [7], but some DMUs may still have slacks as either input excesses, shortfalls in output, or both. So although we consider the CRS frontier to dictate efficiency, it is more effective as a reference.

Though there is no test for the best specification or model in DEA as noted by Berg in 2010, the slacks-based efficiency measure displays a higher discriminating power for measuring efficiency and modeling environmental performance. One aim of this article is to propose a test method for DEA models in order to show the efficacy of our proposed technology called the Latent Variable Model. To clear the way for the introduction of this model the axiomatic proposition of weak disposability of outputs must be limited in its application and some misunderstandings in its application resolved.

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It is curious to note that despite its superior performance, slacks-based efficiency measures were used in only 7% of articles in the 2008 survey above while only 6% used the VRS reference technology exclusively. In 1984 with Banker et al. extended the CRS model to include Variable Returns to Scale, VRS, in their BCC Model [8]. This more accurately reflects business returns to scale RTS and the efficiency frontier that they are constrained by. Whereas the CRS frontier is linear, a straight line (or hyper-line when multiple inputs and outputs are used), and CRS models tend to "linearize" efficiency ratios, the VRS frontier is a convex hull that "envelops" the DMU set. VRS models tend to overstate efficiency since the VRS frontier is easier to achieve, but they also contain more DMUs on their frontier than the CRS frontier. As a result, Variable Returns to Scale give a more accurate and realistic description of a target unit, DMU_0, relative to its peers in terms of the most achievable RTS reference technology. Therefore, to develop our Latent Variable Model we use the VRS frontier as our reference technology.

Though much work characterizes the problem of undesirable outputs, few studies if any, have actually resolved the problem of simultaneously reducing inputs and undesirable outputs while adhering to basic causality in production theory. This study proposes weak disposability of inputs to serve both purposes. Thus, we use a Radial Input Reduction Model as our efficiency measure, which is 'benchmarked' against a comparable SB Model.

To expand DEA's scope of application, this study carefully reorganizes the postulates necessary for an environmental DEA in Section 2. Section 2 will also cover the definition and properties of the Latent Variable model. In Section 3, several models from the literature will be compared to the LV Model for consistency. Section 4 presents the first application study as an example of 'linearization.' Section 5 applies the Malmquist decomposition to the OECD CO_2 emissions change in order to compare the performance of the 7 test models. Section 6 summarizes the findings of this paper.

2. Reference technology and efficiency measure assumptions

An aggregated production technology with undesirable outputs can be formulated causally as $T=[(X,Y,U):X \text{ produces } (Y,U)]$. Consider an aggregated production process across world regions or countries with energy consumption as an aggregate input vector (X_j), with gross domestic product (Y_j) as an aggregate desirable output, and with aggregate energy-related CO_2 emissions (U_j) as an undesirable output vector across j DMUs. An examination of the data from one period to the next shows that U and Y do not move proportionally as outputs [9–12] and for some DMUs they move in opposite directions. In every case, however, X is the cause for both U and Y. This causality condition is the foundation of the production process, that is, inputs (or input mix) create undesirable outputs, and outputs do NOT create undesirable outputs except possibly in physical state changes in industrial applications such as petrochemical refineries. Hence, in this study the weak disposability of inputs is considered rather than the weak disposability of outputs [13–15], which could be considered a special case and not relevant to aggregate data. The dual weak disposability model is explored by Kuosmanen and Matin [16]; this article employs its envelopment implementation.

2.1. Weak disposable inputs and the latent variable

Latent Variable technology uses a Radial Input Model in conjunction with weak disposability applied to the aggregate input vector. This idea was somewhat anticipated by Tone in his development of his epsilon efficiency measure [17]. He also used a unit vector in his Stage II slacks model [18] similar to our benchmark Reverse SBM.

The weak disposability of inputs means that the mix of aggregate inputs, X_{pj}, are reduced by the direct input reduction objective, α as follows:

$$\frac{\sum_{j=1}^{J} z_j x_{pj}}{\alpha_0 x_0} = 1 \tag{1}$$

The input objective Min αunder VRS will make the sum of the z-weighted reference set in the numerator equal to the input objective x_0, thus weakly disposing of inputs proportionally.

Similarly, the Latent Variable is the natural ratio of the sum of the z-weighted reference set of aggregate undesirable outputs, u_{rj}, and the reduction target u_0 as follows:

$$\frac{\sum_{j=1}^{J} z_j u_{rj}}{u_0} \le \lambda_0 \tag{2}$$

The reduction coefficient, λ_0, is a latent ratio implied by the linear programming constraints. The Latent Variable responds to (and thus can keep track of) the simultaneous reduction of undesirable outputs as the direct reduction of inputs takes place according to the model specification. In the case of a reduction LVM, the specification uses weak disposability of inputs (Min α). On the other hand, to reduce undesirable outputs directly (Min λ) under the assumption of weakly disposable outputs, as is the case throughout the literature, often referred to as 'treating outputs as inputs' [19] and even touted as an Environmental Index, ignores the fact that X causes U and that X must be instrumental in U's reduction. The question is: do the results of a Min λ model have any meaningful interpretation?

Note that the Latent Variable is the natural ratio of the undesirable output constraint in the envelopment model. It does not imply that a new variable has been added to the linear programming (LP) model. Therefore, Eq. (3) is a valid LP model and not a Non-Linear Model (NLP). Furthermore, models that 'treat outputs as inputs' by directly reducing U also ignore the effect this has on the Latent Variables within their mis-specified models. That is, they assume inputs are held constant, but this is not the case since the inequality on the input constraint will allow inputs to be reduced! In fact, this anomaly was how the Latent Variable was discovered. Hence, several practitioners have been using a 'mis-specified Latent Variable model' in their research, but due to this misunderstanding, have misinterpreted their results!

Let us correctly specify the Latent Variable model now. To generalize the Latent Variable Input Model suppose two time periods exist, K and L, and that vectors with P inputs, Q outputs, R undesirable outputs, and J DMUs are given as shown in Eq. (3). The equality on the input constraint allows weak disposability of inputs which implies costs may be involved in reducing inputs depending on the mix. This mix for the purposes of calculation is non-separable [9]. The equality here is also similar to the non-proportional inputs of other non-radial models [20]. The constraints are the same as in the standard BCC model, otherwise.

As inputs are directly minimized the value for normal outputs will remain unchanged unless the linear combination of weights dictates an increase, which is only possible if the DMU under consideration is an outlier with very inefficient outputs. Similarly undesirable outputs will remain unchanged unless the linear combination of weights dictates a decrease, which is likely for undesirable outputs since the aggregated data includes a variety of input mixes and CO_2 producing technologies. These inefficiencies are captured in the Latent Variable, λ, which has the same range characteristics as other reduction variables. The convexity constraint ensures Variable Returns to Scale, $\sum_{j=1}^{J} z_j = 1$. Without it, the model will generate Constant Returns to Scale, which is a valuable benchmark for the exploration of Returns to Scale RTS. The CRS LV Min α model will be used later for this purpose.

Hence, the Latent Variable Input Model is correctly specified as VRS LV Min α to indicate the direct minimization of the input vector X on the VRS frontier. Min λ envelopment models are misspecified because they directly minimize the output vector U. These designations, Min α and Min λ, are used throughout to indicate proper and improper specifications, respectively

$$\begin{aligned}
&\text{VRS LV Min } \alpha: \\
&\{\forall \text{DMU} \| j=1,2,\ldots,J;\, t=K,L\} \\
&\text{s.t.} \sum_{j=1}^{J} z_j x_{pj}^t = \alpha x_{p0}^t, \quad p=1,2,\ldots,P \\
&\sum_{j=1}^{J} z_j y_{qj}^t \ge y_{q0}^t, \quad q=1,2,\ldots,Q \\
&\sum_{j=1}^{J} z_j u_{rj}^t \le \lambda u_{r0}^t, \quad r=1,2,\ldots,R \\
&\sum_{j=1}^{J} z_j = 1 \\
&z_j \ge 0 \\
&\text{Latent variable} \quad 0 \le \lambda = \frac{\sum_{j=1}^{J} z_j u_{rj}^t}{u_{p0}^t} \le 1, \\
&\{\forall \text{ DMU} \| r=1,2,\ldots,R;\, p=1,2,\ldots,P\} \qquad (3)
\end{aligned}$$

3. Environmental estimation models

Discrepancies tend to accumulate over time and appear concisely in two articles by Zhou et al. [21] and Zhou and Ang [22]. In the first study, they introduce the Pure and Mixed Environmental Index (PEI and MEI) for the CRS, NIRS and VRS with a Min λ envelopment specification. Mixed radial models have been discussed by Cooper et al. [23], Zhu [19] and others. Zhou, Ang, and Poh introduce their Mixed Environmental model by combining PEI model with a feedback coefficient. The feedback condition setup between the inputs and the weighting sum using the variable β, tends to shift between frontier types, from NIRS to CRS to VRS. Their mixed model, shown in Eq. (4), implied the advantage of 'scale matching' in their results, but the model was not stable under testing due to its Min λ specification and β's influence on the z-weighting factor. This is an un-natural specification in DEA that has no justification or interpretation.

$$\begin{aligned}
&\text{NIRS MEI } = \text{Min } \lambda: \\
&\text{s.t.} \sum_{j=1}^{J} z_j x_{pj} \le \beta x_{p0}, \quad p=1,2,\ldots,P \\
&\sum_{J=1}^{J} z_j y_{qj} \ge y_{q0}, \quad q=1,2,\ldots,Q \\
&\sum_{J=1}^{J} z_j u_{rj} = \lambda u_{r0}, \quad r=1,2,\ldots,R \\
&\sum_{J=1}^{J} z_j = \beta, \quad j=1,2,\ldots J \\
&z_j \ge 0, \quad \beta \le 1 \qquad (4)
\end{aligned}$$

Their second study focused on a Malmquist styled decomposition of CO_2 emissions calling for two reduction variables which they derive from two separate CRS models, one Min α and one Min λ. Designated as the Double CCR model and shown in Eq. (5), it employs their PEI model with the defects assessed above in CCR_1, and a valid radial input model in CCR_2. These independent and uncorrelated reduction coefficients are then used in a Malmquist styled decomposition. Their application studies show several infeasible points on the Efficient Production Frontier, one 'linearized' factor on the frontier, and seven decomposition factors which are discussed in our application studies

$$\begin{aligned}
&\text{CCR}_1 = \text{PEI} = \text{CRS Min } \lambda: && \text{CCR}_2 = \text{CRS Min } \alpha: \\
&\text{s.t.} \sum_{1}^{J} z_j x_{pj} \le x_{p0}, \quad p=1,2,\ldots,P && \text{s.t.} \sum_{J=1}^{J} z_j x_{pj} \le \alpha x_{p0} \\
&\sum_{1}^{J} z_j y_{qj} \ge y_{q0}, \quad q=1,2,\ldots,Q && \sum_{J=1}^{J} z_j y_{qj} \ge y_{q0} \\
&\sum_{1}^{J} z_j u_{rj} = \lambda u_{r0}, \quad r=1,2,\ldots,R, && \sum_{J=1}^{J} z_j u_{rj} = u_{r0} \\
&z_j \ge 0, \quad j=1,2,\ldots,J, && z_j \ge 0 \qquad (5)
\end{aligned}$$

Slack based environmental DEA technologies have been widely studied in the literature [24–31]. The Slacks Based Model (SBM) is an additive non-radial, non-oriented (neither input nor output oriented) approach to variable reduction. For inputs and undesirable outputs, the slacks model removes excesses s_p^- and s_r^-. For regular outputs it adds shortfalls s_q^+. By maximizing the excess and shortfall slacks using an additive objective, the slacks model is the most discriminating in terms of efficiency. Phase II is applied after the reduction coefficients have been obtained from a previous radial or non-radial model. As such, it is more difficult to apply and appears in less than 5% of studies. Phase II is shown as follows:

$$\begin{aligned}
&\text{SBM Phase II } = \text{Max}\left(\sum_{1}^{P} s_p^{t-} + \sum_{1}^{Q} s_q^{t+} + \sum_{1}^{R} s_r^{t-}\right): \\
&\{\forall \text{ DMU} \| j=1,2,\ldots,J;\, t=K,L\} \\
&\text{s.t.} \sum_{1}^{J} z_j x_{pj}^t + s_p^{t-} = \alpha^* x_{p0}^t, \quad p=1,2,\ldots,P \\
&\sum_{1}^{J} z_j y_{qj}^t - s_q^{t+} = y_{q0}^t, \quad q=1,2,\ldots,Q \\
&\sum_{1}^{J} z_j u_{rj}^t + s_r^{t-} = \lambda^* u_{r0}^t, \quad r=1,2,\ldots,R \\
&\sum_{1}^{J} z_j = \rho \\
&z_j \ge 0 \\
&\text{For CRS use } \rho \ge 0 \\
&\text{For VRS use } \rho = 1 \\
&\text{where } \alpha^* \text{ and } \lambda^* \text{ are derived from Phase I} \qquad (6)
\end{aligned}$$

In this study, the SBM is our benchmark for comparison since it exhibits Koopmans Efficiency (KE) rather than the simple Farrell efficiency exhibited on the frontier. Our benchmark was constructed by assuming that all DMUs wind up on the frontier; we call this Reverse SBM.

Suppose a DMU is on both frontiers, that is $\alpha_j^* = 1$ and $\lambda_j^* = 1$. These reduction coefficient vectors are constructed as a Unit Vectors and applied to SBM Phase II. Subsequently, the SBM Phase II model will 'force' all the slacks excesses into s_p^{t-} and s_r^{t-}, and all the shortfalls into, s_q^{t+} for each period. Optimal reduction coefficients are then calculated for each period by removing these slacks from the data and dividing by the appropriate original value such that $\hat{X}_{pj}^t = X_{pj}^t - s_{pj}^{t-*} \to (\hat{x}_0^t / x_0^t) \to \alpha_{Optimal}^t$ or $\hat{U}_{rj}^t = U_{rj}^t - s_{rj}^{t-*} \to (\hat{u}_0^t / u_0^t) \to \lambda_{Optimal}^t$.

So in summary, we have four basic models: our Latent Variable, the Double CCR, the NIRS Mixed Environmental, and our benchmark

Reverse SBM. We will test two version of LV: the VRS and CRS; and we will test two versions of CCR: the original and one using LV but mis-specified as Min λ. As mentioned earlier this mis-specified LV approach has been unwittingly applied by many researchers. Also we present the original study results by Zhou and Ang. Our study uses a sensitive multiplicative decomposition to judge the overall performance of each technology since no method has yet emerged in the literature to compare models of this nature.

3.1. CO_2 decomposition methods in DEA

The Malmquist Productivity Index [19,32–34] has been used throughout the literature to evaluate technology change and its effect on inputs and outputs. It is defined as the maximum factor by which inputs in one period could be reduced to produce the same output in a second period. It can be calculated using either CRS or VRS input technology. Recently it has been used in 26% of the energy and environmental performance reported in the survey above. The following articles are of particular relevance: [21,26,35–37].

Suppose a production technology in period K as reference, then the functional representation of the reduction coefficient for Carbon Emissions is $\lambda_j^K(X_j^L,Y_j^L,U_j^L)$, where the objective values are in period L. We state this more concisely by simplifying the functional notation to $\lambda_j^K(U_j^L)$, which represents the performance measure for CO_2 emissions in period L as calculated with reference to the technology of period K. Similarly, $\alpha_j^K(X_j^L)$ is the performance measure for Energy Consumption in period L as calculated with reference to the technology of period K.

In general, the Malmquist Productivity Index (MPI) itself can be decomposed into two components: Technical Efficiency change and Efficient Production Frontier shift. We combine our functional notation for the application study with their MPI components in Eq. (7). Both the Reduction Objective, α, and the Latent Variable, λ, are reduction coefficients for this decomposition method. When derived from a single integrated LVM the reduction coefficients exhibit relative correlation, which in turn helps to minimize the variance of this model

$$\mathrm{MPI}_X=\left(\frac{(\alpha_j^K(X_j^K)\alpha_j^L(X_j^K))}{(\alpha_j^K(X_j^L)\alpha_j^L(X_j^L))}\right)^{1/2}=\left(\frac{\alpha_j^K(X_j^K)}{\alpha_j^L(X_j^L)}\right)\left(\frac{(\alpha_j^L(X_j^L)\alpha_j^L(X_j^K))}{(\alpha_j^K(X_j^L)\alpha_j^K(X_j^K))}\right)^{1/2}$$

$$\mathrm{MPI}_U=\left(\frac{(\lambda_j^K(U_j^K)\lambda_j^L(U_j^K))}{(\lambda_j^K(U_j^L)\lambda_j^L(U_j^L))}\right)^{1/2}=\left(\frac{\lambda_j^K(U_j^K)}{\lambda_j^L(U_j^L)}\right)\left(\frac{(\lambda_j^L(U_j^L)\lambda_j^L(U_j^K))}{(\lambda_j^K(U_j^L)\lambda_j^K(U_j^K))}\right)^{1/2} \quad (7)$$

Here we refrain from using Shephard [38] input and output Distance functions (SD) as in Zhou and Ang's study since these lead to infeasibility problems on the frontier. To decompose the change in aggregate CO_2 emissions, we start with its essential form [39]. That is, aggregate CO_2 emissions in period L are the product of Carbon Factor (U_j^L/X_j^L), Energy Intensity (X_j^L/Y_j^L), and output GDP. The change over two periods is therefore: $\Delta CO_2=\Delta$Carbon factor $\times$ ΔEnergy Intensity $\times$ ΔGDP.

The Malmquist Productivity Index in Eq. (7) is applied as a reciprocal to form the decomposition in Eq. (8). The decomposition terms of the MPI itself are tagged onto the end of the formula to balance the inverted ratios. According to Zhou and Ang the first term is Potential Carbon Factor Change PCFCH followed by Potential Energy Intensity Change PEICH and GDP change. We are not sympathetic to these terms, but we use them for comparison purposes only. Consult Zhou and Ang for the meanings they assign

$$\frac{U_j^L}{U_j^K}=\left(\frac{(U_j^L/(\lambda_j^K(U_j^K)\lambda_j^L(U_j^K))^{1/2})}{(U_j^K/(\lambda_j^K(U_j^L)\lambda_j^L(U_j^L))^{1/2})}\frac{(1/X_j^L)}{1/X_j^K}\right)$$
$$\times\left(\frac{(X_j^L/(\alpha_j^K(X_j^K)\alpha_j^L(X_j^K))^{1/2})}{(X_j^K/(\alpha_j^K(X_j^L)\alpha_j^L(X_j^L))^{1/2})}\frac{(1/Y_j^L)}{(1/Y_j^K)}\right)\left(\frac{Y_j^L}{Y_j^K}\right)$$
$$\times\left(\frac{\lambda_j^K(U_j^K)}{\lambda_j^L(U_j^L)}\right)\left(\frac{\left(\lambda_j^L(U_j^L)\lambda_j^L(U_j^K)\right)}{\left(\lambda_j^K(U_j^L)\lambda_j^K(U_j^K)\right)}\right)^{1/2}$$
$$\times\left(\frac{\alpha_j^K(X_j^K)}{\alpha_j^L(X_j^L)}\right)\left(\frac{(\alpha_j^L(X_j^L)\alpha_j^L(X_j^K))}{(\alpha_j^K(X_j^L)\alpha_j^K(X_j^K))}\right)^{1/2} \quad (8)$$

The fourth and fifth terms are part of CO_2 Emissions Performance CEPCH: 'catch up effect' of Technical Efficiency Change called CEEFCH, and 'frontier shift' of Carbon Abatement Technology CATECH. Analogously, terms six and seven are part of Energy Usage Performance EUPCH: 'catch up effect' of Energy Usage Technical Efficiency Change EUEFCH, and the 'frontier shift' of Energy Savings Technology ESTECH. The terminology is given below in two forms for easy cross reference

$$\mathrm{CO_2CH}_j=\mathrm{PCFCH}_j\cdot\mathrm{PEICH}_j\cdot\mathrm{GDPCH}_j\cdot\mathrm{CEEFCH}_j$$
$$\mathrm{CATECH}_j\cdot\mathrm{EUEFCH}_j\cdot\mathrm{ESTECH}_j$$
$$\Delta\mathrm{CO_2}=\Delta P\text{ Carbon Factor}\times\Delta P\text{ Energy Intensity}\times\Delta\mathrm{GDP}$$
$$\times\Delta\text{Carbon TE}\times\Delta\text{Carbon Shift}\times\Delta\text{Energy TE}$$
$$\times\Delta\text{Energy Shift} \quad (9)$$

Since an increase in one factor must lead to a decrease in others, the decomposition is a fine proving ground for detecting model inaccuracies. We refer to this as 'multiplicative sensitivity.'

4. Application study 1: Infeasible world regions

The data is from Key World Energy Statistics [40]. The data sets are those used by Zhou and Ang for comparison purposes. The Double CCR study uses Million Tons of Oil Equivalent (Mtoe) for primary energy supply. 1995 Billion PPP USD is used for Gross Domestic Product (GDP) and CO_2 emissions from fuel combustion is measured in million tonnes (Mt). The OECD study uses Petajoules for primary energy.

The data sets, presented in Tables 1 and 4, are ranked by relative carbon emissions change. Relative change is the change in each region or country divided by the total change in the variable over time, given by

$$\text{Relative Change}_{XJ}=\frac{(x_j^L-x_j^K)}{(\sum_{J=1}^{j}x_j^L-\sum_{J=1}^{j}x_j^K)} \quad (10)$$

We include ranked relative change percentages in the data sets to indicate the relative scale of changes that will be taken into consideration within the LP minimization. Thereby outliers and the spectrum of diversities within the data sets become obvious and demonstrates the need for a VRS reference frontier.

Based on Zhou and Ang's decomposition study we compare results under different conditions and using different models. The first study adjusts their world regions study using Latent Variable technology and our hybrid decomposition. This makes the frontier DMUs feasible while identifying unwanted artifact that arises when the CRS frontier is used as the reference technology. In Table 2, their original results, note that Latin America has no feasible values for 4 out of 7 of the decomposition factors, yet this is the frontier reference DMU. A second problem is obvious in CATECH where all the values are the same. Because of the multiplicative sensitivity of the decomposition many of the subsequent values must be incorrect since they compensate for the uni-value in CATECH. To investigate these anomalies we construct an integrated yet mis-specified LVM Min λ from their Double CCR shown in Eq. (5).

Table 1
World regions data set ranked by relative carbon change (%).
Source: Key world energy statistics [40].

World regions	2002			2004			Relative change (%)		
	***X* (Mtoe)**	***Y* (GDP)**	***U* (Mt)**	***X* (Mtoe)**	***Y* (GDP)**	***U* (Mt)**	***X* (Mtoe)**	***Y* (GDP)**	***U* (Mt)**
China	1245	5359	3307	1626	7219	4769	46.1	21.0	61.5
OECD	5346	25,375	12,554	5508	29,493	12,911	19.6	46.4	15.0
Asia	1184	5508	2257	1290	6777	2499	12.8	14.3	10.2
Middle East	431	1026	1093	480	1282	1183	5.9	2.9	3.8
Former USSR	931	1552	2232	979	1989	2313	5.8	4.9	3.4
Africa	540	1669	743	586	1997	814	5.6	3.7	3.0
Latin America	455	2567	845	485	3119	907	3.6	6.2	2.6
Non-OECD Europe	100	358	253	104	413	265	0.5	0.6	0.5
Total	10,232	43,414	23,284	11,058	52,289	25,661	100	100	100

Table 2
Original decomposition results.
Source: Zhou and Ang [22].

Region	CO_2 emissions change and its seven components for world regions, 2002–2004.							
	C^T/C^0	PCFCH	PEICH	GDPCH	CEEFCH	CATECH	EUEFCH	ESTECH
OECD	1.0284	1.1281	0.9462	1.1623	1.0016	0.8834	1.0108	0.9268
Middle East	1.0823	1.1220	0.8662	1.2495	0.9805	0.8834	1.1362	0.9056
Former USSR	1.0363	1.2187	0.8086	1.2816	0.9153	0.8834	1.1205	0.9056
Non-OECD Europe	1.0474	1.1093	0.9139	1.1536	1.0278	0.8834	1.0821	0.9116
China	1.4421	1.0314	1.0540	1.3471	1.2118	0.8834	1.0000	0.9198
Asia (exclusive of China)	1.1072	1.1293	0.9579	1.2304	1.0187	0.8834	1.0024	0.9222
Latin America	1.0734	–	–	1.2150	1.0000	–	1.0000	–
Africa	1.0956	1.1026	0.9628	1.1965	1.0365	0.8834	1.0189	0.9245
Geometric mean[a]	1.1128	1.1190	0.9271	1.2300	1.0242	0.8834	1.0516	0.9166

[a] The calculation of the geometric mean excludes the data for Latin America.

Table 3
Regional decomposition results using the latent variable.
Source: Abraham Bretholt.

DMUs	Integreated CCR model using latent variable and efficiency scores.							
	CO_2CH	**PCFCH**	**PEICH**	**GDPCH**	**CEEFCH**	**CATECH**	**EUEFCH**	**ESTECH**
OECD	1.028	1.128	1.000	1.162	1.002	**0.883**	1.000	**0.877**
Middle East	1.082	1.122	1.000	1.250	0.981	**0.883**	1.021	**0.877**
Former USSR	1.036	1.219	1.000	1.282	0.915	**0.883**	1.042	**0.877**
Non-OECD Europe	1.047	1.109	1.000	1.154	1.028	**0.883**	1.000	**0.877**
China	1.442	1.031	1.000	1.347	1.212	**0.883**	1.054	**0.877**
Asia	1.107	1.129	1.000	1.230	1.019	**0.883**	1.000	**0.877**
Latin America	1.073	1.068	0.937	1.215	1.000	**0.943**	1.000	**0.937**
Africa	1.096	1.103	1.000	1.197	1.036	**0.883**	1.001	**0.877**
Geomean	1.108	1.112	0.992	1.228	1.021	**0.891**	1.015	**0.884**

Eq. (11) preserves the fundamental error of Double CCR: 'treating outputs as inputs' by directly minimizing U. However, this is a Latent Variable model so we keep track of the Latent Variable, α in this case. Now the reduction coefficients, λ and α, are correlated to the same production causality. As a result this integrated LVM exaggerates the CRS LP artifact by also producing 'linearized' uni-values for the Energy Consumption frontier as well. See ESTECH shown in Table 3. This vindicates previous authors who suggest that VRS models are more in keeping with reality than CRS ones [14,41]

$$\text{CRS LV} = \text{Min}\ \lambda:$$
$$\{\forall \text{DMU} \| j = 1,2,\ldots,J;\ t = K,L\}$$
$$\text{s.t.} \quad \sum_{J=1}^{J} z_j x_{pj}^t \le \alpha x_{p0}^t, \quad p = 1,2,\ldots,P$$
$$\sum_{J=1}^{J} z_j y_{qj}^t \ge y_{q0}^t, \quad q = 1,2,\ldots,Q$$
$$\sum_{J=1}^{J} z_j u_{rj}^t = \lambda u_{r0}^t, \quad r = 1,2,\ldots,R$$
$$z_j \ge 0,$$
$$\text{Latent Variable } 0 \le \alpha = \frac{\sum_{J=1}^{J} z_j x_{pj}^t}{x_{p0}^t} \le 1$$
$$\{\forall\ \text{DMU} \| p = 1,2,\ldots,P\} \tag{11}$$

Note the induced distortions in other decomposition values, and also that the Latin America frontier is now feasible. We also note from their OECD study that CATECH exhibits a very low standard deviation of 0.019 (SD) indicating that a degree of linearization must be present.

5. Study 2: Decomposing OECD CO_2 using the latent variable model

Our second study uses the Organization of Economic Development and Cooperation, the OECD, as its focus. The data set is given in Table 4. Notice various combinations of increase and decrease are possible giving evidence to the diverse mix of aggregate inputs and aggregate GDP under varying technologies. That is, in some cases there are increasing GDP and decreasing Energy Consumption and CO_2 Emissions; in some cases the reverse; and in some cases all are increasing, etc. Hence it seems unreasonable to tie undesirable outputs and normal outputs to proportional change as implied by weak disposability of outputs. Nor does it seem reasonable to think that a country with virtually no CO_2 emissions could exist without income (GDP) as implied by the null-joint lemma. Hence, careful thinking must be applied to causality within the data set before axioms are applied carte blanche.

Decomposition results using the VRS Min α Latent Variable Model are shown in Table 5. Values less than one are considered improvements, that is, contributing to the reduction of CO_2 emissions over the period. Since Technical Efficiency is measured in CEEFCH and EUEFCH, if these values are equal to one, then the DMU is on the Efficient Production Frontier. A good example is France, on the EPF where a shift in Energy Saving Technology ESTECH and reduction in Potential Carbon Factor PCFCH has led to the reduction of CO2CH. On the other hand, Potential Energy Intensity PEICH and GDP change have counteracted the reduction of CO2CH. Other DMUs can be interpreted similarly. Also observe that all DMUs and factors are feasible under the VRS Min α Latent Variable Model and that the values show incremental change since the period under consideration is only one year. Sweden, however, 'fell off' the VRS frontier during this period which caused an abrupt change in its Energy Usage Efficiency factor, EUEFCH. Sweden's 'falling off' the frontier will be explored in the Scale Efficiency analysis to follow.

In Table 6 a comparison is made to Zhou and Ang's results in terms of factor reversals. That is, if a factor in our study shows improvement and the same factor in their study shows deterioration, then this is a factor reversal. We also show infeasible factors as reversals as in the case of Italy and Switzerland. Another example, France has four reversals and it is on the VRS frontier as are five other DMUs, shown in the 'On EPF' column. The 'Low SD' column shows whether the standard deviation of the factors for each DMU is lower in the LVM. Lower variability indicates a 'smoother' transition from period to period.

Overall results show 72 of 180 possible reversals (40%); this result overturns the previous analysis. Standard deviation is lower for 20 of 30 DMUs (67%) indicating less abrupt changes from period to period. The Latent Variable Model also determined that 8 DMUs populate the VRS frontier. As expected there are more DMUs on the non-CRS frontier, which contributes to the reversals on the frontier in CEEFCH and EUEFCH. Thus, our integrated production theoretic LVM results reflect a more accurate assessment of the factors contributing to CO_2 reduction in the OECD for 2001–2002. That is, since the integrated and correlated derivation of the reduction coefficients is consistent with the production assumptions of Data Envelopment Analysis, our results reflect the actual causal reductions possible within the data set (Table 7).

Table 8 presents a statistical summary of the decomposition results by model. As noted, CO2CH and GDPCH remain unchanged in each model. Other decomposition factors promote a reduction in CO_2 emissions if their values are less than one; conversely if their values are greater than one, they contribute to an increase in CO_2 emissions.

Table 4
OECD data set ranked by relative carbon change (%).
Source: Key world energy statistics [40].

OECD	2001			2002			Relative change (%)		
	X (Pt J)	*Y* (GDP)	*U* (C Mt)	*X* (Pt J)	Y GDP	*U* (C Mt)	*X* (Pt J)	*Y* (GDP)	*U* (C Mt)
Japan	21,646	3038	1165	21,643	3042	1207	0.1	1.0	46.0
United States	94,366	8978	5614	95,895	9196	5652	73.8	47.8	41.8
Spain	5352	767	287	5508	783	303	7.5	3.4	17.5
Canada	10,390	817	521	10,468	843	532	3.8	5.8	11.6
Korea	8119	675	442	8520	718	452	19.3	9.4	10.8
Turkey	2997	379	185	3158	409	193	7.8	6.5	8.6
Italy	7226	1334	426	7231	1338	433	0.2	1.1	7.7
Mexico	6366	812	360	6586	820	365	10.6	1.6	5.7
Portugal	1065	163	59	1105	163	63	1.9	0.2	4.2
Finland	1418	124	61	1491	127	64	3.5	0.6	3.3
Sweden	2143	223	49	2137	227	50	0.3	0.9	1.7
Australia	4536	479	342	4719	492	343	8.8	2.9	1.1
Luxembourg	161	19	8	169	19	9	0.4	0.0	1.0
New Zealand	758	74	33	754	77	34	0.2	0.7	0.8
Greece	1202	170	90	1215	177	91	0.6	1.4	0.3
Netherlands	3235	407	178	3263	408	178	1.4	0.2	0.2
Iceland	141	8	2	143	8	2	0.1	0.0	0.1
Denmark	838	136	52	827	139	51	0.5	0.6	0.4
Ireland	634	109	43	641	117	43	0.3	1.6	0.7
Norway	1107	127	34	1110	128	33	0.1	0.3	0.7
Hungary	1071	118	56	1066	122	56	0.2	0.9	0.8
Switzerland	1173	200	44	1136	200	43	1.8	0.1	1.2
Austria	1292	209	67	1275	212	66	0.8	0.6	1.3
Slovak Republic	772	54	39	776	57	38	0.2	0.5	1.5
Czech Republic	1733	136	119	1747	139	115	0.7	0.6	3.9
Belgium	2470	253	120	2382	254	113	4.2	0.4	7.6
France	11,152	1436	384	11,132	1453	377	1.0	3.8	7.8
Poland	3770	367	292	3734	372	283	1.7	1.1	9.3
United Kingdom	9814	1374	542	9483	1398	529	16.0	5.2	13.5
Germany	14,795	1935	850	14,501	1938	838	14.2	0.8	13.7

Table 5
Decomposition results for the VRS Min α Latent Variable Model.
Source: Abraham Bretholt.

OECD	Final decomposition of CO_2 change for OECD countries for 2001–2: Radial Min X LV							
	CO_2CH	PCFCH	PEICH	GDPCH	CEEFCH	CATECH	EUEFCH	ESTECH
Australia	1.0029	0.9863	0.9997	1.0273	0.9797	0.9977	1.0244	0.9888
Austria	0.9822	0.9953	1.0029	1.0139	1.0000	1.0000	1.0023	0.9683
Belgium	0.9415	1.0434	0.9999	1.0067	0.9573	0.9774	0.9803	0.9773
Canada	1.0205	1.0240	0.9998	1.0326	0.9818	1.0075	0.9817	0.9942
Czech Republic	0.9696	1.0083	0.9993	1.0191	1.0085	0.9459	1.0336	0.9577
Denmark	0.9922	1.0194	1.0007	1.0206	1.0306	0.9570	1.0103	0.9564
Finland	1.0496	0.9693	0.9991	1.0226	1.0956	0.9400	1.0791	0.9538
France	0.9813	0.9830	1.0044	1.0121	1.0000	1.0000	1.0000	0.9820
Germany	0.9852	1.0226	1.0006	1.0018	0.9587	1.0253	0.9808	0.9969
Greece	1.0033	1.0222	0.9989	1.0376	1.0119	0.9596	1.0091	0.9665
Hungary	0.9875	1.0342	0.9986	1.0348	1.0240	0.9369	1.0121	0.9517
Iceland	1.0476	1.0092	1.0235	1.0000	1.0000	1.0235	1.0000	0.9909
Ireland	0.9861	1.0130	0.9687	1.0686	1.0000	0.9628	1.0000	0.9767
Italy	1.0167	1.0079	0.9970	1.0037	1.0000	1.0080	1.0000	1.0000
Japan	1.0363	1.0181	1.0111	1.0014	1.0000	1.0181	1.0000	0.9875
Korea	1.0224	1.0114	0.9995	1.0634	0.9587	1.0048	0.9945	0.9927
Luxembourg	1.1071	1.0070	1.0170	1.0106	1.0000	1.0474	1.0000	1.0212
Mexico	1.0144	0.9752	0.9999	1.0091	0.9982	1.0073	1.0314	0.9940
Netherlands	1.0011	0.9937	1.0000	1.0025	1.0056	0.9932	1.0201	0.9864
New Zealand	1.0210	1.0456	0.9935	1.0431	1.0391	0.9447	0.9941	0.9656
Norway	0.9822	0.9795	1.0023	1.0095	1.0000	1.0000	1.0274	0.9645
Poland	0.9705	1.0229	0.9998	1.0139	0.9671	0.9905	0.9921	0.9848
Portugal	1.0660	0.9732	0.9992	1.0049	1.0981	0.9613	1.0723	0.9637
Slovak Republic	0.9644	1.0348	0.9913	1.0444	0.9602	0.9655	0.9923	0.9785
Spain	1.0560	0.9910	0.9999	1.0205	1.0287	1.0065	1.0151	0.9936
Sweden	1.0330	1.0359	0.9596	1.0193	1.0000	1.0000	1.5872	0.6423
Switzerland	0.9749	1.0067	0.9840	1.0020	1.0000	1.0000	1.0000	0.9822
Turkey	1.0427	1.0188	0.9990	1.0778	0.9788	0.9923	0.9927	0.9858
United Kingdom	0.9771	1.0593	1.0091	1.0172	0.9421	1.0133	0.9458	0.9952
United States	1.0069	0.9981	0.9899	1.0243	1.0000	0.9927	1.0000	1.0021

Table 6
Decomposition results compared to literature: factor reversals.
Source: Abraham Bretholt.

DMUs	Factor reversals using VRS Min α LV compared to literature							
	PCFCH	PEICH	CEEFCH	CATECH	EUEFCH	ESTECH	Low SD	OnEPF
Australia						True	True	
Austria	True			True	True		True	
Belgium					True		True	
Canada			True	True		True	True	
Czech Republic			True			True		
Denmark		True	True					
Finland		True						
France	True						True	True
Germany		True	True	True				
Greece			True			True		
Hungary			True		True			
Iceland	True	True	True	True			True	True
Ireland					True		True	True
Italy	True	True	True	True	True	True	True	True[a]
Japan			True	True			True	True
Korea				True	True	True	True	
Luxembourg	True			True	True	True	True	True
Mexico		True	True	True			True	
Netherlands							True	
New Zealand			True					
Norway	True		True					
Poland					True	True	True	
Portugal		True						
Slovak Republic					True	True	True	
Spain		True		True			True	
Sweden			True	True			True	
Switzerland		True	True	True	True	True	True	True[a]
Turkey					True		True	
United Kingdom		True		True				
United States	True				True		True	True

[a] Infeasible DMUs on the EPF from the literature are feasible on the LV frontier.

The models as established earlier are the benchmark Slacks Model, Rev SBM, then our proposed Latent Variable Model, VRS Min α. The next model violates the production process and 'treats outputs as inputs,' VRS Min λ with slacks removed was modified from its CRS version which was used to exaggerate the artifact in Study 1. CRS Min α is an LVM used to derive Scale Efficiency and Returns to Scale. The Double CCR derives its reduction coefficients from two different models and the decomposition does not use Shepard Distances. Finally Zhou and Ang's result are presented.

All models concur in some aspects: the countries of the OECD, on average, are promoting lower carbon levels through Energy Saving and Carbon Abatement Technology and by reduced Potential Energy Intensity. That is, that factor averages of ESTECH, CATECH, and PEICH are in each case less than 1. Also the Malmquist Productivity Indices for Carbon Reduction, CTFP MI, a gage of Total Factor Productivity, indicates advance. This is what we would expect and indicates that all models are responding to the decomposition in a similar way: there are no gross errors in the methodology of the decomposition or the models.

The models are ranked by lowest LAV Score, designed to capture the Least Aggregate Variability (LAV). Little variation is expected in decomposition factors over a short period, so exaggerated values due to an inaccurate model can be easily detected by Range Checking. This procedure takes the difference between the maximums and

Table 7
Model rangesum scores and reduction coefficient correlations.
Source: Abraham Bretholt.

	Rev SBM	VRS Min α	CRS Min α	Dbl CCR[a]	Zhou and Ang	VRS Min λ[b]	NIRS MEI[b]
Least aggregate variability (LAV)							
RC Correl	0.8167	0.8350	0.6098	0.0178	na	0.4103	0.2697
Add Inv	0.1833	0.1650	0.3902	0.9822	1.0000	0.5897	0.7303
RangeSum	1.4000	1.6840	1.7050	2.3590	2.3730	4.5610	7.4820
StDev GM	0.2700	0.3270	0.3360	0.4160	0.4300	1.0050	1.4010
LAV Score	0.4107	0.4496	0.6069	0.9878	1.0068	1.3930	1.9708
Rank	1	2	3	4	5	6	7

[a] No Shepard distances.
[b] Slacks removed.

Table 8
Statistical summary of model variance.

	CO_2CH	PCFCH	PEICH	GDPCH	CEEFCH	CATECH	EUEFCH	ESTECH	CTFP MI	ETFP MI
Decomposition statistical summary by model										
Rev SBM										
Geomean	1.007	1.020	1.007	1.022	1.001	0.978	0.994	0.986	0.979	0.980
Minimum	0.941	0.953	0.968	1.000	0.979	0.875	0.941	0.769		
Maximum	1.107	1.206	1.270	1.078	1.040	1.018	1.026	1.081		
StDev	0.036	0.051	0.051	0.020	0.014	0.028	0.017	0.053		
VRS Min α										
Geomean	1.007	1.010	0.998	1.022	1.000	0.989	1.022	0.967	0.989	0.988
Minimum	0.941	0.969	0.960	1.000	0.942	0.937	0.946	0.642		
Maximum	1.107	1.059	1.024	1.078	1.098	1.047	1.587	1.021		
StDev	0.036	0.023	0.012	0.020	0.035	0.028	0.109	0.064		
VRS Min λ										
Geomean	1.007	1.018	0.960	1.022	1.000	0.981	1.093	0.940	0.981	1.027
Minimum	0.941	0.964	0.494	1.000	0.934	0.905	0.607	0.669		
Maximum	1.107	1.113	1.422	1.078	1.091	1.094	2.519	1.651		
StDev	0.036	0.034	0.187	0.020	0.034	0.034	0.444	0.216		
NIRS MEI No Slk										
Geomean	1.007	1.010	1.007	1.022	1.025	0.965	1.290	0.759	0.989	0.980
Minimum	0.941	0.651	0.558	1.000	0.719	0.655	0.608	0.376		
Maximum	1.107	1.205	1.371	1.078	2.341	1.186	3.403	1.299		
StDev	0.036	0.088	0.170	0.020	0.257	0.078	0.554	0.198		
CRS Min α										
Geomean	1.007	1.012	0.991	1.022	0.977	1.010	1.010	0.986	0.987	0.995
Minimum	0.941	0.963	0.781	1.000	0.911	1.000	0.949	0.754		
Maximum	1.107	1.057	1.006	1.078	1.081	1.013	1.663	0.999		
StDev	0.036	0.027	0.040	0.020	0.039	0.005	0.125	0.044		
Dbl CCR No ShD										
Geomean	1.007	1.023	0.980	1.022	0.998	0.979	1.019	0.987	0.977	1.006
Minimum	0.941	0.962	0.781	1.000	0.754	0.928	0.971	0.754		
Maximum	1.107	1.395	1.063	1.078	1.118	1.005	1.663	1.021		
StDev	0.036	0.074	0.046	0.020	0.061	0.011	0.122	0.046		
ZA result										
Geomean	1.007	1.027	0.980	1.022	0.998	0.974	1.019	0.987	0.973	1.006
Minimum	0.942	0.962	0.781	1.000	0.754	0.891	0.971	0.754		
Maximum	1.107	1.395	1.064	1.078	1.118	0.982	1.663	1.021		
StDev	0.036	0.078	0.047	0.020	0.061	0.019	0.122	0.047		

minimums given in Table 8 for each factor. The RangeSum is then taken over the factors for each model. The RangeSums and geometric means of the SDs (StDev GM) are combined with the additive inverse (Add Inv) of the reduction coefficients correlations (RC Correl) by taking their GM to form the LAV Score. The Reverse SBM, VRS Min α and CRS Min α exhibit the Least Aggregate Variability.

In Table 7 models ranked 4 through 7 are Min λ and show the poorest performance. The CRS and VRS Latent Variable Models perform well and are used for Scale Efficiency tests. The Reverse SBM serves as the benchmark for this study since it has no slacks to perturb its accuracy and is, thereby, Koopmans Efficient.

5.1. Scale efficiency comparisons

As previously explained, Scale Efficiency is an integral part of DEA that cannot be ignored during the model building process. It has been demonstrated that the Latent Variable Model has integrated Multiple Objectives, α and λ, and performs well compared to the SBM benchmark. Another advantage of the LVM is that both reduction coefficients can be used to estimate scale efficiency. Thus both consumption-side and carbon-side evaluations of scale can be made. Such a comparison is impossible without an LVM and its inherent properties.

In Table 9 the OECD countries are organized according to their distribution along the VRS frontier from decreasing returns (DRS) through constant returns (CRS) to increasing returns (IRS). The countries on the CRS frontier exhibit the Most Productive Scale Size, MPSS [42]. For 2001 and 2002 the MPSS countries were Italy and Switzerland, whereas Sweden was also included in 2001, but became slightly inefficient on the consumption side in 2002 and "fell off" the CRS frontier: its Energy Intensity efficiency declined to 95.4% in 2002.

Also in Table 9 note that Sweden, Norway, Iceland and France demonstrate Scale Efficiency in carbon emissions reduction (SE $U=1$), but show inefficiencies in energy consumption reduction (SE $X<1$). Comparisons can be made against the relative changes given in the data set as a method to cross check the LV decomposition results. For example, the USA is very inefficient in terms of Scale Efficiency on both sides (about 51.7%). It now is obvious that the US devours 73.8% of the OECDs new Energy Consumption because of its gross Scale Inefficiency. In addition this Scale Inefficiency caused huge shortfalls (35.2%) in GDP in 2002 since US Energy Consumption translated into only 47.8% of the OECDs new income, not 73.8%. Similarly Japan's Scale Inefficiency (about 82.3%) is responsible for 46.0% of the OECDs rise in Carbon Emissions.

Figs. 1 and 2 plot the GM of the Scale Efficiency of the reduction coefficients for 2001–2002. Fig. 1 excludes DMUs on the VRS frontier since these are shown to be outliers in Fig. 2. From Fig. 1 it is evident that DMUs are conforming to certain standards by the trend of their reduction pattern (slope is roughly $=4.08$). Outside of the CRS efficient DMUs Italy and Switzerland at (1,1), four countries are performing better than the trend to reduce their emissions: Sweden, Norway, Austria and England. DMUs in Fig. 2 show a different trend in energy consumption and emissions (slope is roughly $=1.15$). These are the DMUs on the VRS frontier, designated by '-F', and they dispose of Carbon Emissions roughly four times slower than the DMUs in Fig. 1 for a given reduction in Energy Consumption. The USA, Germany, Japan and France all show Decreasing Returns to Scale (DRS) which shapes this trend.

Table 9
Scale efficiencies using both reduction coefficients.
Source: Abraham Bretholt.

OECD	Scale efficiency							
	2001		2002		Consumption side		Emissions side	
	RTS	*Z* Sum	RTS	*Z* Sum	SE $X-01$	SE $X-02$	SE $U-01$	SE $U-02$
United States	DRS	6.733	DRS	6.871	0.516	0.518	0.511	0.527
France	DRS	4.264	DRS	4.702	0.728	0.726	1.000	1.000
Japan	DRS	2.278	DRS	2.273	0.761	0.759	0.834	0.816
Germany	DRS	1.451	DRS	1.448	0.851	0.851	0.900	0.889
Sweden	CRS	1.000	DRS	1.075	1.000	0.954	1.000	1.000
United Kingdom	DRS	1.030	DRS	1.044	0.984	0.977	0.990	0.983
Italy	CRS	1.000	CRS	1.000	1.000	1.000	1.000	1.000
Switzerland	CRS	1.000	CRS	1.000	1.000	1.000	1.000	1.000
Canada	IRS	0.612	IRS	0.630	0.996	0.999	0.987	0.993
Mexico	IRS	0.609	IRS	0.613	0.996	0.999	0.987	0.993
Spain	IRS	0.575	IRS	0.585	0.995	0.999	0.985	0.992
Korea	IRS	0.506	IRS	0.536	0.994	0.999	0.980	0.990
Norway	IRS	0.383	IRS	0.415	0.948	0.964	1.000	1.000
Australia	IRS	0.359	IRS	0.368	0.989	0.997	0.964	0.980
Turkey	IRS	0.284	IRS	0.305	0.984	0.997	0.950	0.974
Netherlands	IRS	0.305	IRS	0.305	0.986	0.997	0.955	0.974
Poland	IRS	0.275	IRS	0.278	0.984	0.996	0.948	0.970
Austria	IRS	0.157	IRS	0.254	0.956	0.987	0.992	1.000
Belgium	IRS	0.189	IRS	0.190	0.974	0.994	0.918	0.952
Greece	IRS	0.128	IRS	0.132	0.959	0.990	0.875	0.928
Portugal	IRS	0.122	IRS	0.122	0.955	0.989	0.879	0.921
Denmark	IRS	0.102	IRS	0.104	0.948	0.987	0.845	0.907
Czech Republic	IRS	0.102	IRS	0.104	0.948	0.987	0.845	0.907
Finland	IRS	0.093	IRS	0.095	0.942	0.986	0.831	0.898
Hungary	IRS	0.088	IRS	0.091	0.939	0.985	0.823	0.894
Ireland	IRS	0.082	IRS	0.087	0.934	0.984	0.810	0.890
New Zealand	IRS	0.056	IRS	0.058	0.892	0.928	0.800	0.860
Slovak Republic	IRS	0.041	IRS	0.042	0.849	0.872	0.788	0.830
Iceland	IRS	0.021	IRS	0.017	0.308	0.296	1.000	1.000
Luxembourg	IRS	0.014	IRS	0.014	0.633	0.607	0.715	0.661

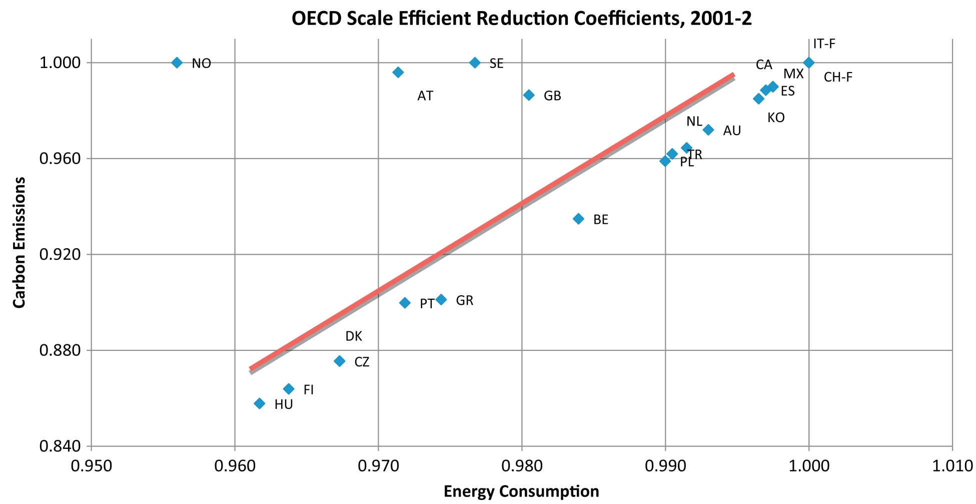

Fig. 1. Scale efficiency trends in the OECD.

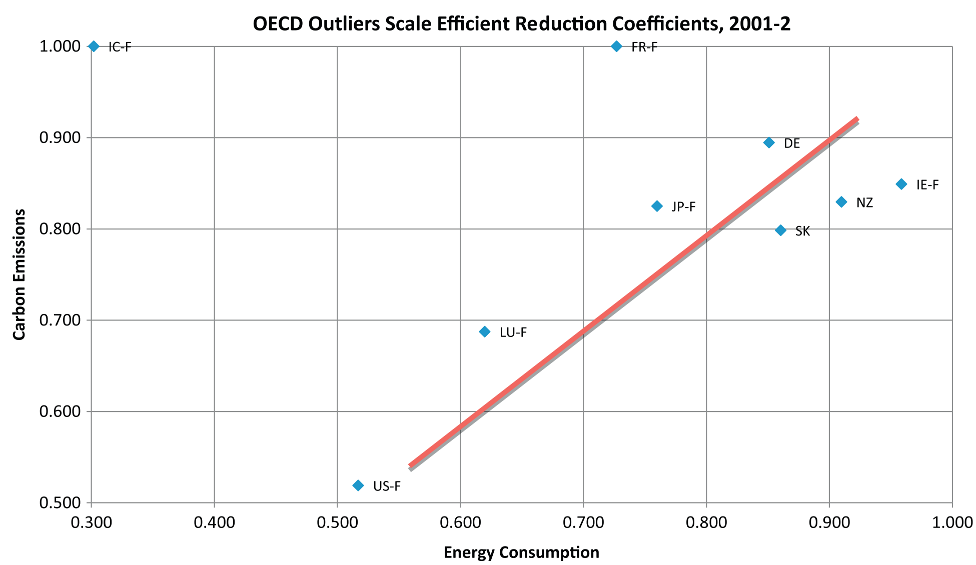

Fig. 2. Scale efficiency of outliers on the OECD frontier.

Furthermore, and of considerable theoretical importance, is that most VRS frontiers are populated by a majority of outliers, as shown here, that are not Scale Efficient. Iceland and France are partial exceptions since both have mitigated emissions to a great extent due to their input mix, geothermal [43] and nuclear respectively. Nevertheless, further research is necessary to unravel this VRS conundrum.

The obvious trends in Scale Efficiency supported by the underlying Factor Reversals in the decomposition and the logical associations with Relative Change in the data set demonstrate some of the numerous advantages of the Latent Variable Environmental Model.

6. Concluding remarks

This article has presented several models that reduce undesirable outputs directly using weak disposability of outputs as their axiomatic basis. For aggregated data it has been proposed that weak disposability of inputs conforms more closely to production theoretic assumptions. Thus the Latent Variable DEA model offers a consistent basis as an environmental technology. Its primary strength is that as a multiple objective decision model in an integrated environment its reduction coefficients are strongly correlated to production causality. As a result both reduction coefficients yield scale efficient outcomes that optimize the informational content of the data. The model is also highly correlated to its slacks based counterpart, used here as a benchmark for model comparison, showing that the LV environmental technology may well be an optimal, slacks-free, Koopmans Efficient model. Further research is necessary to confirm this point.

In this study a new method for model comparison has also been devised to evaluate the efficacy of ‘treating outputs as inputs’ since this is often the interpretation given to weak disposability of outputs in the literature. The application studies use a Malmquist styled decomposition to test the variance between models. This type of model comparison has not been done before: it proposes that the range sum and standard deviation of decomposition factors are sensitive indicators of model performance. Also it proposes that reduction coefficient correlation gauges the relative adherence of a model to production causality. When combined, these elements confirm that Min λ models are flawed from the production theoretic perspective when aggregate data is under consideration.

The results of the application studies show that the VRS reference technology is superior to the CRS frontier in decomposition studies since the later tends to linearize the frontier factors of the Malmquist Index. The results of the new VRS Latent Variable model applied to the same OECD study from the literature shows factor reversals in 40% of cases. That is, if a factor was shown to decrease under the CRS uncorrelated scenario, it was shown to increase under the integrated VRS LV environmental model. Finally, two graphs presenting the Scale Efficiency of the simultaneous reduction of Carbon Emissions and Energy Consumption demonstrate the efficacy of Latent Variable analysis since such correlated results have not been possible before. The outliers on the second graph reveal that the VRS frontier also has its shortcomings, that is, although the frontier is Farrell efficient many of its DMUs show a high degree of scale inefficiency. This reveals an important discrepancy in Data Envelopment Analysis that warrants further research.

Overall the introduction of the Latent Variable Data Envelopment Analysis technology is a first step towards the analysis of not only undesirable outputs, but for the inclusion of externalities that impact both business and society. Carbon emissions were ignored for many decades before they became an important theme in Data Envelopment Analysis. This study by using the integrated Latent Variable Reduction model, has demonstrated the production theoretic simultaneous reduction of Carbon Emissions through their causal linkage with Energy Consumption. Similarly much research lies ahead for the inclusion of other externalities using the Latent Variable Method.

References

[1] Zhou P, Ang BW, Poh KL. A survey of data envelopment analysis in energy and environmental studies. European Journal of Operational Research 2008;189:1.

[2] Rahmstorf S. Climate change—state of the science. Potsdam Institute for Climate Impact Research, 2008.

[3] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. European Journal of Operational Research 1978;2:429.

[4] Charnes A, Cooper WW, Golany B, Seiford LM, Stutz J. Foundations of data envelopment analysis for Pareto-Koopman's efficient empirical production functions. Journal of Econometrics 1985;30:1.

[5] Koopmans T. Activity analysis of production and allocation. New York: John Wiley & Sons; 1951.

[6] Pareto V. Manuel d'Economie Politique. Paris: Giars & Briere; 1909.

[7] Farrell MJ. The measurement of productive efficiency. Journal of the Royal Statistical Society 1957;120:253.

[8] Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science 1984;30:1078.

[9] Fare R, Grosskopf S, Noh D, Weber W. Characteristics of a polluting technology: theory and practice. Journal of Econometrics 2005;126:469.

[10] Färe R, Grosskopf S, Lovell CAK, Pasurka C. Multilateral productivity comparisons when some outputs are undesirable: a nonparametric approach. Review of Economics and Statistics 1989;71:90.

[11] Sueyoshi T. Stochastic DEA for restructure strategy: an application to a Japanese petroleum company. Omega International Journal of Management Science 2000;28:385.

[12] Liu WB, Zhang DQ, Meng W, Li XX, Xua F. A study of DEA models without explicit inputs. Omega International Journal of Management Science 2012;40:65.

[13] Färe R, Grosskopf S. Modeling undesirable factors in efficiency evaluation: Comment. European Journal of Operational Research 2004;157:242.

[14] Scheel H. Undesirable outputs in efficiency valuations. European Journal of Operational Research 2001;132:400.

[15] Seiford LM, Zhu J. Modeling undesirable factors in efficiency evaluation. European Journal of Operational Research 2002;142:16.

[16] Kuosmanen T, Matin RK. Duality of weakly disposable technology. Omega International Journal of Management Science 2011;39:504.

[17] Tone K, Tsutsui M. An epsilon-based measure of efficiency in DEA. In: C. R. I. o. E. P., editor. Industry. GRIPS Policy Information Center; 2001. Discussion Paper:09-13. Retrieved from 〈http://r-center.grips.ac.jp/gallery/docs/09-13_new.pdf〉.

[18] Tone K. A slacks-based measure of efficiency in data envelopment analysis. European Journal of Operational Research 2001;130:498.

[19] Zhu J. Quantitative models for performance evaluation and benchmarking. Boston: Kluwer Academic Publishers; 2004.

[20] Fare R, Lovell CAK. Measuring technical efficiency. Journal of Economic Theory 1978;19:150.

[21] Zhou P, Poh KL, Ang BW. A non-radial DEA approach to measuring environmental performance. European Journal of Operational Research 2007;178:1.

[22] Zhou P, Ang BW. Decomposition of aggregate CO_2 emissions: a production-theoretical approach. Energy Economics 2008;30:1054.

[23] Cooper WW, Seiford LM, Tone K. DEA: a comprehensive text with model, applications, references, and DEA solver software. New York City: Springer; 2007.

[24] Hu JL, Kao CH. Efficient energy-saving targets for APEC economies. Energy Policy 2007;35:373.

[25] Hu J-L, Wang S-C. Total-factor energy efficiency of regions in China. Energy Policy 2006;34:3206.

[26] Ramanathan R. An analysis of energy consumption and carbon dioxide emissions in countries of the Middle East and North Africa. Energy Journal 2005;30:2831.

[27] Sueyoshi T, Goto M. Slack-adjusted DEA for time series analysis: performance measurement of Japanese electric power generation industry in 1984–1993. European Journal of Operational Research 2001;133:232.

[28] Zhu J. Setting scale efficient targets in DEA via retums to scale estimation method. Journal of the Operational Research Society 2000;51:376.

[29] Zofio JL, Prieto AM. Environmental efficiency and regulatory standards: the case of CO_2 emissions from OECD industries. Research in Energy Economics 2001;23:63.

[30] Tone K, Tsutsui M. Dynamic DEA: a slacks-based measure approach. Omega International Journal of Management Science 2010;38:145.

[31] Juo J-C, Fu T-T, Yu M-M. Non-oriented slack-based decompositions of profit change with an application to Taiwanese banking. Omega International Journal of Management Science 2012;40:550.

[32] Caves DW, Christensen LR, Diewert WE. Multilateral comparisons of output input and productivity using index numbers. Economic Journal 1982;92:73.

[33] Fare R, Grosskopf S, Lovell CAK. Production frontiers. Cambridge: Cambridge University Press; 1994.

[34] Malmquist S. Index numbers and indifference surfaces. Trabajos de Estatistica 1953;4:209.

[35] Boyd GA, Tolley G, Pang J. Plant level productivity efficiency and environmental performance of the container glass industry. Environmental and Resource Economics 2002;23:29.

[36] Kumar S. Environmentally sensitive productivity growth: a global analysis using Malmquist–Luenberger index. Ecological Economics 2006;56:280.

[37] Murillo-Zamorano LR. The role of energy in productivity growth: a controversial issue? Energy Journal 2005;26:69.

[38] Färe R, Grosskopf S. Intertemporal production frontiers, with dynamic DEA. Boston: Kluwer Academic Publishers; 1995.

[39] Zhou P, Ang BW. Decomposition of aggregate CO_2 emissions: a production-theoretical approach. Energy Economics 2008;30:1054.

[40] International-Energy-Agency. International Energy Agency key world energy statistics. Paris: OECD; 2004, 2006.

[41] Färe R, Grosskopf S. Modeling undesirable factors in efficiency evaluation: comment. European Journal of Operational Research 2004;157:242–245.

[42] Banker RD. Estimating most productive scale size using DEA. European Journal of Operational Research 1984;17:35.

[43] Tester JW. The future of geothermal energy: impact of enhanced geothermal systems (EGS) on the United States in the 21st century.Massachusetts Institute of Technology; 2006.